Chapter – Probability

Probability is all about the chance that something will happen. What is the chance that it will rain today? If there are no clouds in the sky it is highly unlikely that it will rain, however, if there are dark grey thunderclouds in the sky there is a much better chance that it will rain today.

We use probability to work out the odds or chance that something will happen. Casinos work out the chance of a gambler winning (low) vs the chance that the casino will win (high). In this way the casino is assured that they will always make money. If the odds were reversed Casino's wouldn't last very long.

Section A – Outcomes and Relative Frequency

An event is the occurrence of something, for example rolling a dice.

An outcome is the result of something. For example, if you rolled a dice [you can do this on your calculator by pressing 2ndF RANDOM 7 and selecting dice by pressing 1] you would have an option of 6 outcomes: 1, 2, 3, 4, 5 or 6.

The Relative frequency is the number of times an outcome occurs e.g. 5, over a number of events (36). Thus your relative frequency is $\frac{5}{36}$. The more times an event occurs, the closer the relative frequency gets to the probability of that event occurring.

Activity 1

1. Say whether the following events are definite, likely, equally possible, unlikely, or impossible:

   a) changing into a frog
   b) winning the world cup (soccer)
   c) being born a girl or a boy
   d) winning the lotto
   e) blinking
   f) rain in the summer
   g) your birthday being today
   h) win a race between 2 people.

2. Given below is the number of outcomes as well as the number of total events they were part of. Determine the relative frequency.

a) O = 60 E = 776
b) O = 625 E = 1 565
c) O = 4 E = 78
d) O = 21 E = 50
e) O = 6 E = 7

3. Look at the relative frequencies for question 2 and state whether they are represent a highly likely, likely, equally possible, unlikely or highly unlikely event.

4. Look carefully at results of a dice thrown below and answer the questions that follow:

6; 4; 3; 1; 3; 6; 6; 4; 1; 4; 5; 6;
3; 3; 2; 1; 6; 3; 6; 5; 6; 5; 1; 2.

a) Determine the relative frequency of the outcome: 6.
b) Determine the relative frequency of the outcome: 2.
c) If you were to bet would you choose 6 or 2 as your bet, and explain why you say so.

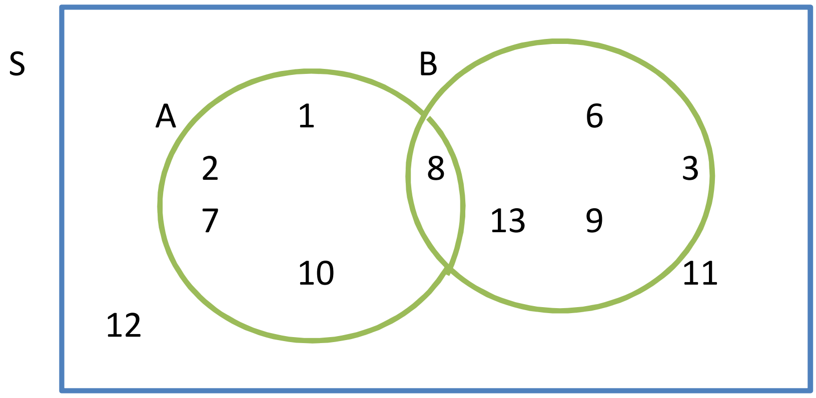
5. Look carefully at the results of a coin tossed below and answer the questions that follow:

H H H T T H H H H H H H
T T T H T H H T H T T T

a) Determine the relative frequency of Heads.
b) Determine the relative frequency of Tails.
c) If you were to choose heads or tails which would you choose and why?
d) Use your calculator (press 2ndF 7 2) to record another 20 throws of the coin (1 = H and 0 = T).
Give the relative frequency of heads and tails now.

Section B- Venn Diagrams

A Venn diagram shows the relation of different events to one another in a set sample space. The sample space (S) is all the possible outcomes that could occur.



Look at the above example.
S is the sample space. It covers the whole of the rectangle. In other words the sample space S = { 1; 2; 3; 6; 7; 8; 9; 10; 11; 12; 13}.
The probability of getting a result in the sample space [P(S)] is always 1. If that outcome is not in the sample space (for example 14) then the probability of it occurring is zero (or impossible) because it is not an option.

There are two events, A and B (the circles) – this means that something happened and the results (or outcomes) are the numbers that occur in the circle.
1, 2, 7, 8 and 10 all occur in event A, while 3, 6, 8, 9, 13 all occur in event B.
11 and 12 occur in neither of the two events.

Can you see that 8 occurs in both event A and B? This is the place where the events overlap. This is called the intersection (∩) – it happens in both A and B. An intersection only occurs where A and B overlap with each other.
Thus the probability of A and B [or P(A and B) or P(A ∩ B)] is the probability of event A happening times the probability of event B happening. So the formula is:

$$P(A \cap B) = P(A) \times P(B)$$

If you wanted the union (∪) of A and B it means all those values in either A or B or both. A nice way to remember union is to think of a union and management making negotiations, either the union or the management or both will win. Thus the probability of A or B [or P(A or B) or P(A ∪ B)] is the probability of event A happening plus the probability of event B happening minus the probability of event A and B happening (because you have already included that outcome in the individual events A and B). So the formula is:

$$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

However, if the events (or in our example circles A and B) did not overlap – the events would be considered mutually exclusive. They are called disjoint events because they do not touch each other. If something is exclusive it means that it is very hard to be a part of that group – they don't let anyone in. So if an event is mutually exclusive it is not part of another event. That means that the formula for the probability of event A or B will change and now becomes:

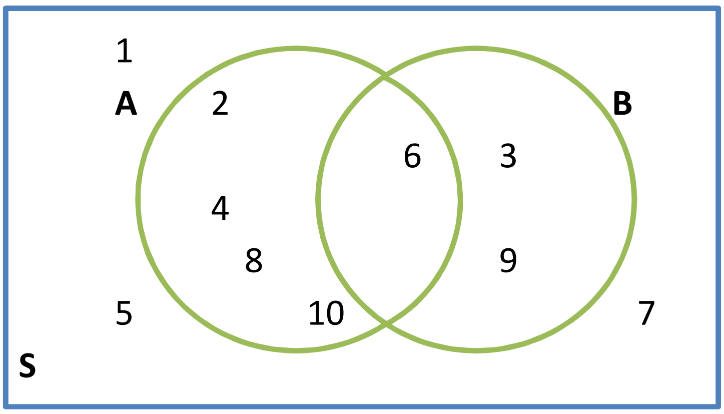
$$P(A \cup B) = P(A) + P(B)$$

Complementary events are events where one outcome occurs and NOT the other. For example the probability of not getting the event A [P(A')] in the above example is the probability of getting all the other outcomes in the sample space or the probability of getting the entire sample space minus the probability of getting the event A. Thus the formula becomes:

$$P(A') = 1 - P(A)$$

Activity 2

1. Study the Venn diagram below very carefully and then answer the questions that follow:



Given that Event A is any even number between 1 and 10, and event B is any multiple of 3, answer the following questions:

a) Are events A and B mutually exclusive? Explain your answer.
b) List the sample space?
c) Give the probability of picking a number from event A.
d) Give the probability of picking a number from event B.
e) What is the probability of selecting a number from A or B?
f) What is the probability that you will select a number that occurs in both event A and B?
g) What is the probability of picking a number that does not occur in event A?
h) What is the probability of selecting a number that occurs in neither Event A nor B?

2. Read the following information carefully then answer all the questions that follow.
A marketing company does a fast food survey and asks 500 people what they order on the side with their takeaway meals.
They found that 71 people only choose ice-cream, while 78 people only choose French fries. Finally, 84 people only choose soda. They also found that 38 people choose ice-cream and soda but not French fries, 17 people choose soda and French fries but not ice-cream, and 22 people choose French fries and ice-cream but not soda. The marketing company also found that 25 people do not choose any sides.

a) Draw a Venn diagram and enter all the given information.
b) Using the Venn Diagram determine the number of people who order all three sides with their takeaways.
c) What are the chances that a person orders ice-cream or French fries or both ?
d) What are the chances that a person will order at least one of the sides?
e) What is the probability that a person will order both French fries and soda?
f) What is the probability that a person will order no sides at all?

3. In a particular school subject choices for the 170 grade 9's are analysed and the following is found: 86 students took maths, 92 took biology, 51 students took science, 20 students took maths and biology, 17 students took maths and science and 6 students took all three subjects, and every student had at least one subject of maths, science or biology.

   a) Draw a Venn diagram to display this information.
   b) Using the Venn diagram determine the number of students who took biology and science but not maths.
   c) What is the probability of picking a student that has chosen maths as one of their subjects?
   d) What is the probability of choosing a student that has chosen both science and maths but not biology?
   e) What is the chance of choosing a student that does not have maths, science or biology?
   f) What is the probability of choosing a student that has chosen either maths or science?

4. A marketing company asks 300 people what movies they have watched in the last 4 weeks. The marketing company finds that there are 3 movies that are the most popular: *James Bond, Madagascar 3* and *Mission Impossible 4.* They find that 45 people have watched all three movies, while 32 people have watched *James Bond* and *Madagascar 3* only. x people have watched *James Bond* and *Mission Impossible 4* only. 21 People watched *Madagascar* and *Mission Impossible 4* only. 108 people watched only *Madagascar,* 148 people watched *James Bond* and 124 people watched *Mission Impossible 4.* Every person interviewed watched at least one of the three movies in the last 4 weeks.

   a) Draw a Venn diagram showing the above information.
   b) Use the information in the Venn diagram to work out the value of x, the number of people that watched both *James Bond* and *Mission Impossible 4.*
   c) *What is the possibility of choosing a person that has watched at least* two of the 3 movies?
   d) What is the probability of choosing someone who has not watched any of the 3 movies?

e) What is probability of choosing someone who has watched either *Mission Impossible 4* or *Madagascar* but not *James Bond?*

f) What is the probability of choosing someone who has watched both *Mission Impossible* and *Madagascar*?

5. Clicks runs a computer program on their system to determine what kind of products shoppers tend to buy together. They monitor 3 products: tissues, nail varnish and shower gel. They find that out of a total of 850 shoppers, 35 shoppers did not buy any of the three products, while 76 bought all three of the products. 106 Shoppers bought both nail varnish and tissues and not shower gel while x shoppers bought nail varnish and shower gel, and x shoppers bought shower gel and tissues. 261 shoppers bought only nail varnish, 240 shoppers bought only tissues and $x + 21$ shoppers bought only shower gel.

a) Draw a Venn diagram using the above information.

b) Determine the value of x.

c) What is the probability of choosing a shopper that has bought all three products?

d) What is the probability of choosing a shopper that has bought at least one of the monitored products?

e) What is the probability of choosing a shopper that has bought at least two of the products?

f) What is the probability of choosing a shopper that has not bought any of the three monitored products?

g) What is the probability of choosing a shopper that has bought both nail varnish and tissues?

h) What is the probability of choosing a shopper that has bought both nail varnish and shower gel?

i) What is the probability of choosing a shopper that has not bought any nail varnish?

<u>C. Dependent and Independent Events</u>

Independent events are events that do not depend on another event to happen, for example, whether you have blue eyes or brown eyes does not depend on the weather.

Dependent events do depend on the previous event to happen, for example, how well you do depends on how well you studied before the exam. Or another example, if you have a pocket full of sweets made up of 10 orange and 5 green sweets, and you pick out a sweet – the chance that you pick an orange sweet is $\frac{10}{15}$ and the chance that you pick out a green sweet is $\frac{5}{15}$. When you choose the next sweet the probability of choosing a green sweet or orange sweet will change depending on what colour you picked the last time.
If you picked an orange sweet previously, then the probability of you picking an orange sweet again is $\frac{9}{14}$ while the probability that you pick a green sweet is $\frac{5}{14}$.
If you picked a green sweet previously, then the probability of you picking an orange sweet is $\frac{10}{14}$ while the probability of picking a green sweet again is $\frac{4}{14}$.
OR $\rightarrow \frac{\textit{how many you have left of that particular item}}{\textit{how many there are left in total}}$

<u>Activity 3</u>

Determine whether the following events are dependent or independent.

a) Event A: a chance of rain on Sunday
Event B: your baby brother having red hair.

b) Event C: surfing the internet
Event D: connecting to the internet

c) Event E: being tired
Event F: watching a late-night movie.

d) Event G: rolling a 6 on the dice
Event H: rolling a 5 on the dice.

D. Two- Way Contingency Tables

A two-way contingency table gives you the values of the outcomes of an experiment or event. For example:

	Men	Women	Total
# of people who got more than 50	99	226	325
# of people who got less than 50	96	79	175
Total	195	305	500

Now you can work out the probability of each occurrence occurring in the general population.

For example, what is the probability of selecting a man who scored above 50?
Answer: there are 99 men who scored more than 50 out of a total of 500.
Thus: $\frac{99}{500} = 0{,}198$

What is the probability of selecting a woman who scored less than 50?
Answer: there are 79 women who scored less than 50 out of a total of 500 women,
Thus: $\frac{79}{500} = 0{,}158$

What is the probability of selecting a woman from the group?
Answer: there are 305 women in the group of 500
Thus: $\frac{305}{500} = 0{,}61$ or 61% chance of choosing a woman from the group.

What is the probability of choosing a man from the group?
Answer: There are 195 men in the group of 500
Thus: $\frac{195}{500} = 0{,}39$

Look at the last two answers. Do you see that they add up to one, or 100%? That is because they are ***mutually exclusive*** *– they* ***cannot occur at the same time****. In our example you cannot be a man and a woman, only a man OR a woman.*

These are also complementary events – if you are not a man, then you must be a woman. This is expressed in the formula: P(M)' = 1 – P(M). In other words, the probability of not being a man is 1 minus the probability of being a man (which adds up to the probability of being a woman).

Another thing that you can use the contingency table for is to work out whether an experiment worked or not. You do this by determining whether your expected values and your observed values are equal (or approximately equal).

Expected and Observed Values

If you were to roll a dice 102 times, you would expect the probability of landing on a certain number to be $\frac{1}{6}$ for all the values on the dice. The more you throw the dice the more likely you are to reach that expected $\frac{1}{6}$ probability. This is the expected outcome – *the probability you would expect if the events were independent.* However if your dice was loaded a certain value may come up more often and the observed probability of the event would be different to the expected probability. If this is the case then the events are not independent and there is some factor coming into play. When statisticians want to see whether an experiment worked or had the desired effect they compare the observed and expected probabilities. If they are the same it means that the experiment didn't work or that the two factors they were comparing did not have an effect on each other. BUT if the expected probability and the observed probability is different it means that the two factors being compared did have an effect on each other. So, if there is a difference between the observed probability and the expected probability it means that the experiment worked.

From our example above:
If you take the probability of being male **and** the probability of scoring over 50 (this is your expected value) you get:

- $P(male).P(score\ over\ 50) = \frac{195}{500} \times \frac{325}{500} = \frac{507}{2\ 000} = 0{,}2535$

And the probability of being a man and scoring over 50 (this is your observed value), you get:

- $P(male\ and\ score\ over\ 50) = \frac{99}{500} = 0{,}198$

You can see that these two values do not match at all. This means that the experiment worked and there is a difference between how many women and how many men scored over or below fifty. You don’t need to worry about how big the difference is and what it means because that is covered at a university level ☺

Activity 4

1. Your teacher conducts an experiment to see whether having higher marks in maths makes you more observant. She sticks up a new poster on the wall in the front of her class and counts how many students notice the poster. She writes down whether the student is in the top half or the bottom half of her class. These are her final results:

Maths Results	**Noticed Poster**	**Didn’t Notice Poster**	**Total**
Top	30	15	45
Bottom	20	25	45
Total	50	40	90

a) If you picked a student at random, what is the probability that you choose a student in the top half of the class that didn’t notice the poster?

b) What is the probability of choosing a student who is in the bottom half of the class and did notice the poster?

c) What is the expected value of the top students who noticed the poster?

d) What is the observed value of the top students who noticed the poster?

e) Do you think that your teacher’s experiment worked? Say why you think so.

2. A glue company wants to check the strength of its glue against another brand's glue strength. They stick the same heavy object to a piece of paper and measure how long it takes to fall off the paper. They put the results into three categories. These are their results:

Length of time in hours	**Glue Company**	**Competitor**	**Total**
8 – 9	*a*	258	1065
6 – 7	135	58	193
4 – 5	58	*b*	742
Total	1000	1000	*c*

a) Determine the values of *a, b* and *c* in the table.

b) What is the probability of purchasing glue from the glue company that will hold for between 6 and 7 hours?

c) What is the probability of purchasing glue from the competitor that will hold for between 6 and 7 hours?

d) What is the expected value for the number of sheets of paper that will hold glue for 4 – 5 hours for the glue company?

e) What is the observed value for the number of sheets of paper that will hold glue for 4 – 5 hours for the glue company?

f) Do you think the glue company's brand of glue is better or worse than the competitor's brand of glue? Give a reason for your answer.

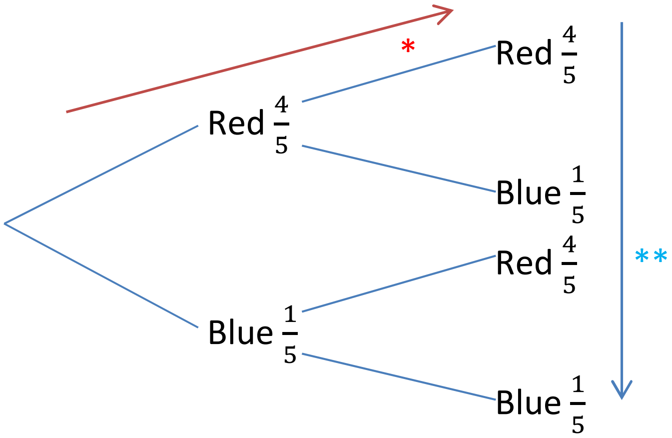
3. A mad scientist is trying to create green eggs and wants to determine if a certain chemical he used has any effect on the egg's colour. After testing 40 different eggs and measuring the intensity of the green with or without the chemical he gets these results:

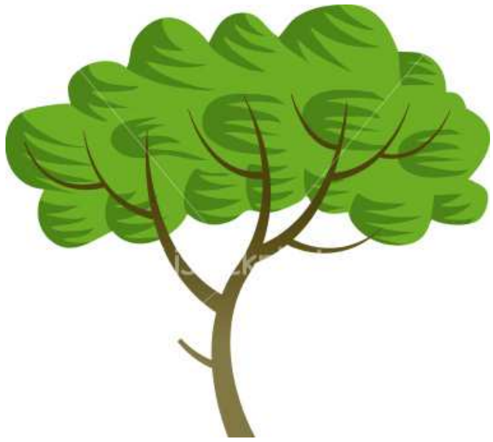
	Intensity below 50	**Intensity above 50**	**Total**
With Chemical	5	15	20
Without Chemical	17	3	20
Total	22	18	40

Do you think the experiment worked? Show all your working out.

E- Tree Diagrams

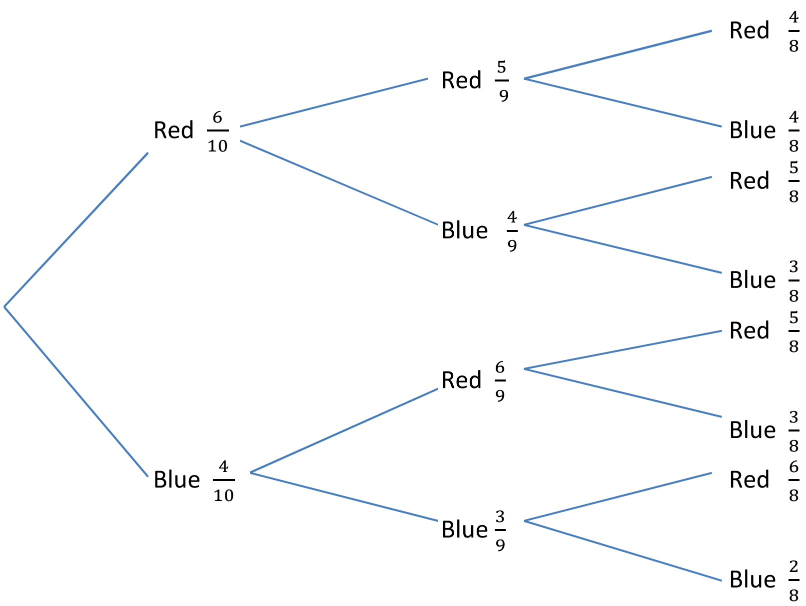
Another great way to work out probability is to use tree diagrams. Tree diagrams start at one point and split into different branches giving the appearance of a tree.





Look at the tree diagram above. You can see that the events are independent because the probability of the first event does not affect the probability of the second event. You can also see that the probability of drawing two red items on a row will be $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$. The general rule when moving along a branch is to multiply*. The probability of drawing a red and blue or blue and red item is $\frac{4}{5} \times \frac{1}{5} + \frac{1}{5} \times \frac{4}{5} = \frac{8}{25}$. When you move down the branches of a tree diagram** you add the probabilities.

When events are dependent the second probability is affected by the results of the first event. For example, if you had 10 marbles in a bag and 6 of them are red and 4 of the marbles are blue, draw a tree diagram to show the probability of selecting either a blue marble or a red marble on the third draw if you do not replace the marble you took out on the previous draw. Your tree diagram would look like this



Do you see that for the draw after the first draw the probability changes – and is dependent on the previous event. The same rules apply – if you want to find the probability going along the tree (keyword: **AND**) then you multiply; if you want to find the probability for something going down the tree (keyword: **OR**) then you would add.

For example: *What is the probability of drawing three blue marbles in a row?*

Keyword: AND ∴ P(3 blue marbles in a row) $= \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} = \frac{1}{30}\ or\ 0{,}0333$

What is the probability of drawing at least two blue marbles in the three rounds of draws?

Keyword: OR ∴ Look for the rows that have at least two blues in it (that means two or more blue marbles), and work out those probabilities, then add them together.
P(at least 2 blues in three rounds)

$$= \frac{6}{10} \times \frac{4}{9} \times \frac{3}{8} + \frac{4}{10} \times \frac{6}{9} \times \frac{3}{8} + \frac{4}{10} \times \frac{3}{9} \times \frac{6}{8} + \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8}$$
$$= \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{30}$$
$$= \frac{1}{3}\ or\ 0{,}3333 \ldots$$

<u>Activity 5</u>

1. *In a class of 30 people there are 12 girls, 17 boys and 1 teacher. There are three prizes to be won in a random draw. If you have won a prize you cannot go back into the draw for another prize.*

   a) Draw a tree diagram to represent this information.
   b) Use your tree diagram to calculate the following:

      i) The probability that only boys will win prizes
      ii) The probability that at least one girl will win a prize.
      iii) The probability that the teacher will win a prize.
      iv) The probability that the teacher will not win a prize.
      v) The probability that 2 girls and one boy will win a prize.

vi) The probability that the first prize will be given to a boy, the second prize to a girl, and the third prize to a teacher.

2. *You have 4 black pens and 7 blue pens in your space-case. After you use the pen you put it back in the space-case before choosing another pen.*

a) Draw a tree diagram to represent the probabilities of the different pens that you use if you use a new pen three times.

b) Use the tree diagram to calculate the following:

i) The probability of choosing only black pens.
ii) The probability of choosing at least one blue pen.
iii) The probability of choosing two blue pens and one black pen.
iv) The probability of choosing a black pen, and then two blue pens in that order.

3. *In your sister's cupboard she has three pairs of pants and two skirts, 4 t-shirts and 3 smart shirts, 8 high heels and 7 pairs of boots. If she wears either a pair of pants or a skirt, either a t-shirt or a smart shirt and either a pair of high heels or a pair of boots, draw a tree diagram to show her different outfit options and then determine the probability that:*

a) Your sister wears a pair of pants a t-shirt and a pair of boots.
b) Your sister wears a skirt, a smart shirt and a pair of high heels.
c) Your sister wears a pair of pants and either a smart shirt and high heels OR a t-shirt and heels.
d) Your sister wears either pants or a skirt and a smart shirt with either a pair of boots or high heels.
e) Your sister does not wear a skirt or a smart shirt.
f) Your sister does not wear a t-shirt and a pair of pants.

4. You go to the flower shop and you choose three different packets of seeds, each packet contains 12 seeds in it. What are the chances that the first three seeds that bud are from the same packet if they are all planted at the same time?

F - Fundamental Counting Principal

The fundamental counting principal determines how many different ways two or more events can occur. For example: if you look at the last question in activity ?.5 you can see that your sister has 3 pairs of pants, 4 t-shirts and 7 pairs of boots. How many different combinations can she make up using her clothing?

- *Thus, every pair of pants (and there are 3) goes with every t-shirt (there are 4) – this means that there are 12 different combinations for the pants and t-shirts or 3 × 4 combinations.*
- *You now have twelve possible combinations and 7 different pairs of boots to wear with each of the twelve combinations, that is 12 × 7 = 84 different combinations.*

You can see from the above example that you multiply the given number of items or events in order to determine how many different combinations you can make.

Thus the fundamental counting principal states that to find the number of different ways two or more events / items can occur together you need to multiply the one event by the other.

Activity 6

Determine the number of ways these items / events can be combined:

1. An option of 3 different milkshakes, and 2 different sides.
2. An option of 3 different milkshakes, 2 different sides and 4 different desserts.
3. An option of 6 different movies, luxury or standard chairs, small, medium or large popcorn, and a slush or soda.
4. Chocolate, vanilla or fudge sponge cake and coffee, cappuccino, tea or hot chocolate.
5. 6 coloured pens, 12 crayons, 4 pencils and 3 oil pastels.

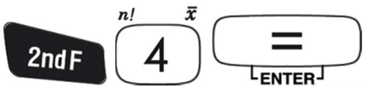
Combinations

Combinations occur when there are a number of different items / events that have to fill a certain number of positions.
For example: Your teacher asks three girls, Betty, Suzy and Bridget to stand in front of the class. How many different ways can the girls stand next to each other if there is a first, second and third position?

If Betty is first then Suzy can be second and Bridget third (1st combination), or Bridget second and Suzy third (2nd combination).
If Suzy is first then Betty could be second and Bridget third (3rd combination), or Bridget could be second and Betty third (4th combination).
If Bridget is first then Betty can be second and Suzy third (5th combination), or Suzy can be second and Betty third (6th combination).
These are all the different possible combinations you can have for these three girls.

This is the same as saying that there are 3 options for the first position, 2 options for the second position and only one option for the third position; OR $3 \times 2 \times 1 = 6$.

When you multiply descending numbers with each other as we just did in this example we call it a factorial, for example 3 factorial is 6. Mathematically we write 3 factorial as 3! or n factorial as n! and so on.

On your SHARP EL-W535 HT calculator you can work out a factorial without having to type for example $6 \times 5 \times 4 \times 3 \times 2 \times 1$ (which is great for bigger numbers). To find 6! simply press 6 

and the answer should be 720.

This is great if you have six positions and six items to fill them – but what happens when you have ten items and only three positions to fill and you cannot repeat any of the items?
The answer is that you use this formula:

$$\frac{n!}{r!(n-r)!}$$

Where *n* is the number of items that you can choose from and *r* is the number of positions you can fill.

Thus from the previous question: $\frac{10!}{3!(10-3)!} = 120$ different combinations without repetition. You can also use your calculator to answer this question (you still have to know the formula but you can use your calculator to check your answer).

Simply put the number of items first then press [2ndF] [5 (nCr)] and then the number of positions to fill. From our previous example you would type 10 [2ndF] [5 (nCr)] 3 [= ENTER] and you would get 120.

What happens if you can repeat an item?
For example – if there are three different milkshakes (chocolate, vanilla and strawberry) and five members of your family, how many different orders of milkshake can you order? The old formula won't work because it doesn't allow for repetition, so we need a new formula which is:

$$\frac{(n+r-1)!}{r!(n-1)!}$$

where *n* is the number of options available and *r* is the number of positions you would to fill. Thus:

$\frac{(3+5-1)!}{5!(3-1)!} = 21$ different combinations where the items can be repeated.

Note: You cannot use your calculator for combinations with repetition

Permutations

In combinations it did not matter in what order the items were placed, however, in permutations the order does matter and this means that there are a lot more ways in which something can be ordered.

For example: you need a 4 digit password for your computer, you can use the digits 0 – 9 and they can be repeated. How many different possible combinations can you make?
Because order does matter in this question we know that it is a permutation and for the first position there 10 possibilities, for the second position there are 10 possibilities, for the third position there are 10 possibilities and for the fourth position there are 10 possibilities, thus your answer will be
$10 \times 10 \times 10 \times 10 = 10^4 = 10\,000$ different combinations.

This gives us another formula: if order does matter and we can repeat items then the number of permutations is $= n^r$ when *n* is your number of options and *r* is the number of positions available.

What happens if you can't repeat the item?
For example – you have 7 different letters in scrabble and you need to make a four letter word to reach a high scoring tile. For your first letter position you have 7 options, the next you have 6 options, the third you have 5 options and the fourth you have 4 options, thus your answer is $7 \times 6 \times 5 \times 4 = 840$
Or you could also use the formula:

$$\frac{n!}{(n-r)!}$$

where *n* is the number of items and *r* is the number of positions to fill.
Let's check if this if this works: $\frac{7!}{(7-4)!} = 840$
Another way to check your answer is to use your EL-W535 HTB SHARP calculator and press 7 2ndF 6 (nPr σx) 4 = (ENTER) your answer will also be 840.

<u>Activity 7</u>

1. *If you have 5 letters and 3 spaces to put them in, determine*

   a) The number of combinations you can make where order doesn't matter and
      i) The letters can be repeated
      ii) The letters cannot be repeated

   b) The number of combinations you can make where order **does** matter and
      i) The letters can be repeated
      ii) The letters cannot be repeated.

2. You go to a restaurant and can choose what toppings you would like on your burger. You have an option of chicken, beef or rib and then you can add any three of the following: lettuce, mayonnaise, cucumber, gherkins, tomato, cheese, pineapple, bacon, egg, and tomato sauce. How many different burgers can you make with these toppings?

3. *You and your 10 friends have a waffle evening, each person is asked to bring a topping*.

a) How many different toppings are possible if each friend does not know what the other friend is bringing?

b) if you are each allowed to put four different toppings on your waffle and if you can use each topping once, how many different combinations of toppings can you have on your waffle?

4. You have a raffle competition in your class and 20 students enter. There are 3 prizes, the first prize is an i-pad, the second prize is a gift voucher to spend at the mall near your school worth R1000 and the third prize is a restaurant voucher of R300. Determine how many different ways a student can win a prize if a student cannot win more than one prize.

5. Your gran is playing bingo and she wants to win the 5-in-the row prize. If there are 50 numbers in bingo and they cannot be repeated, what is the probability that out of the first three numbers called your gran will have 3 of them in her row?

6. You have a friend with Obsessive Compulsive Disorder who likes to put his pens into a certain order. If he has five different coloured pens and three spaces to fill, how many different combinations can he make, and what is the probability that you guess the correct order of his pens and their colour?

7. Your friend tells you that he can guess what you had for dinner the night before if you can tell him what the last 5 meals consisted of. You had a different vegetable every, a different starch every night and a different meat every night. What is the probability that he guesses exactly what you had for dinner the night before.

Answers for Activities

Activity 1

1. a) impossible
   b) unlikely
   c) equally possible
   d) highly unlikely
   e) definite
   f) likely
   g) highly unlikely $\left(\frac{1}{365}\right)$
   h) equally possible

2. a) $\frac{60}{776} = \frac{15}{194}$
   b) $\frac{625}{1565} = \frac{125}{313}$
   c) $\frac{4}{78} = \frac{2}{39}$
   d) $\frac{21}{50}$
   e) $\frac{6}{7}$

3. a) highly unlikely (0,08)
   b) unlikely (0,4)
   b) highly unlikely (0,05)
   d) unlikely (0,42)
   e) highly likely (0,86)

4. a) Outcome 6 = 7 Events = 24

   ∴ relative frequency = $\frac{7}{24}$

   b) Outcome 2 = 2 Events = 24

   ∴ relative frequency = $\frac{2}{24}$ or $\frac{1}{12}$

   c) 6, it has a much higher relative frequency

5. a) Outcome Heads = 14 Events = 24

   ∴ relative frequency = $\frac{14}{24}$ or $\frac{7}{12}$

   b) Outcome Tails = 10 Events = 24

   ∴ relative frequency = $\frac{10}{24}$ or $\frac{5}{12}$

   c) Heads, higher relative frequency

d) This will depend on your own results. Remember to include all the previous values so that your number of events will be 44.

Activity 2

1. a) No they are not mutually exclusive because the circles overlap, that is, you can have an even number that is also a multiple of 3; like 6.

b) S {1; 2; 3; 4; 5; 6; 7; 8; 9; 10}

c) P(A) = $\frac{5}{10} = \frac{1}{2}$

d) P(B) = $\frac{3}{10}$

e) P(A or B) = P(A) + P(B) – P(A and B)

$= \frac{1}{2} + \frac{3}{10} - \frac{1}{10}$

$= \frac{7}{10}$

f) P(A and B) = $\frac{1}{10}$

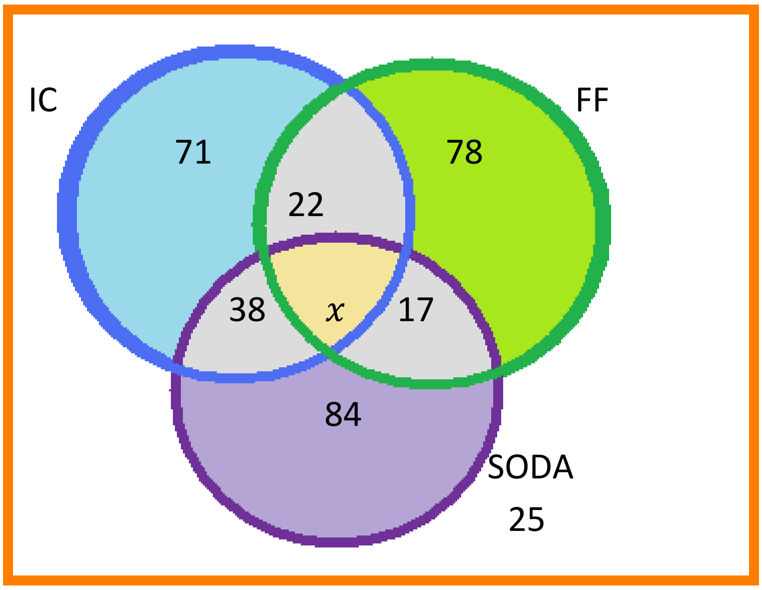
g) P(A') = 1 – P(A)

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

h) P(A' or B') = $\frac{3}{10}$

2. a) S



b) x = 500 – 71 – 78 – 22- 38 – 17 – 84 – 25

= 165 choose all three sides

c) P(IC or FF) = P(IC) + P(FF) – P(IC and FF)

$$= \left(\frac{71+22+38+165}{500}\right) + \left(\frac{78+22+165+17}{500}\right) - \left(\frac{22+165}{500}\right)$$

$$= \frac{296}{500} + \frac{282}{500} - \frac{187}{500}$$

$$= \frac{391}{500}$$

d) P(ordering at least 1 side) = 1 – P(not ordering any sides)

$$= 1 - \frac{25}{500}$$

$$= \frac{19}{20}$$

e) P(FF and SODA) = $\frac{165+17}{500}$

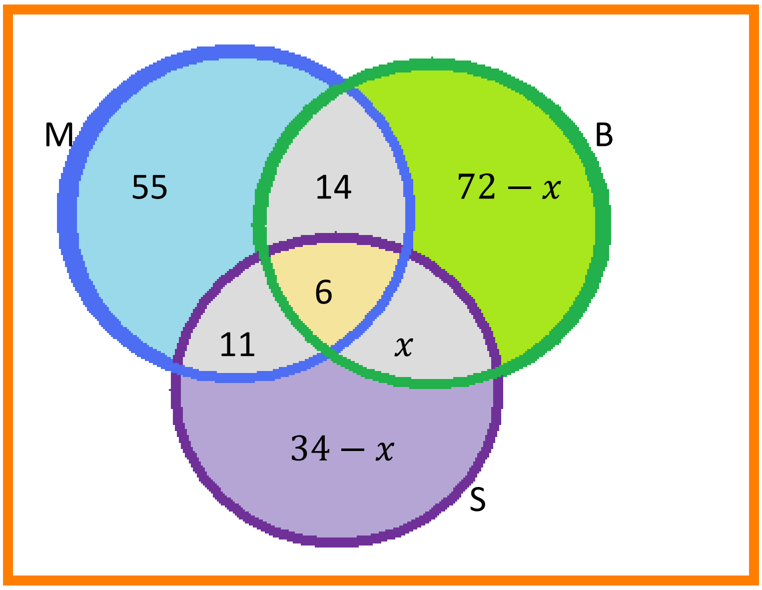
$$= \frac{182}{500}$$

$$= \frac{91}{250}$$

f) P(no sides at all) = $\frac{25}{500}$

$$= \frac{1}{20}$$

3. a) S



b) $170 = 55 + 14 + 72 - x + 6 + 11 + x + 34 - x$

$\therefore 170 = 192 - x$

$\therefore x = 22$

c) P(M) $= \frac{55+14+11+6}{170}$

$= \frac{86}{170}$

$= \frac{43}{85}$

d) P (M and S) $= \frac{11}{170}$

e) P(not have M, S, or B) = 0

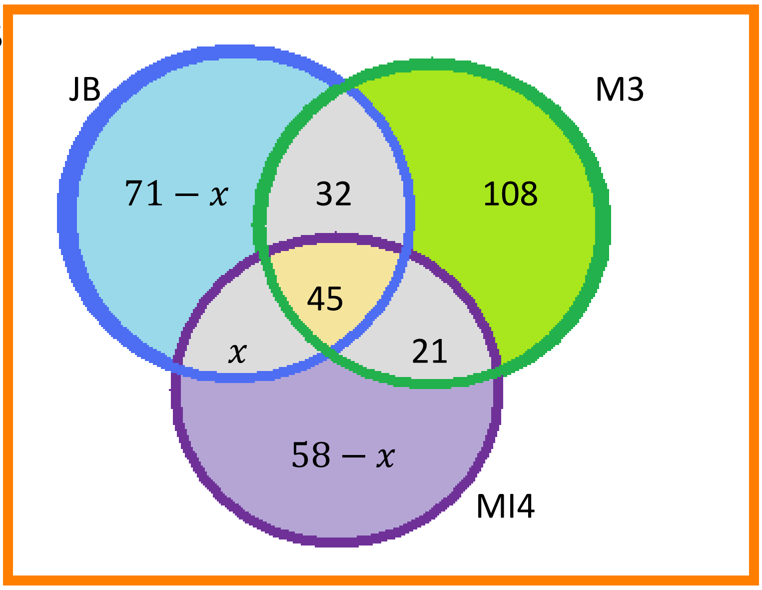
f) P(M or S) = P(M) + P(S) – P(M and S)

$= \frac{43}{85} + \frac{12+11+6+22}{170} - \frac{11+6}{170}$

$= \frac{43}{85} + \frac{51}{170} - \frac{17}{170}$

$= \frac{12}{17}$

4. a) S



b) $300 = 71 - x + 32 + 108 + x + 45 + 21 + 58 - x$

$\therefore 300 = 335 - x$

$\therefore x = 35$

c) P(at least two movies) $= \frac{35}{300} + \frac{32}{300} + \frac{21}{300} + \frac{45}{300}$

$= \frac{133}{300}$

d) P(no movies) = 0

e) P(MI4 or M3 but not JB) = $\frac{108}{300} + \frac{21}{300} + \frac{23}{300}$

$= \frac{152}{300}$

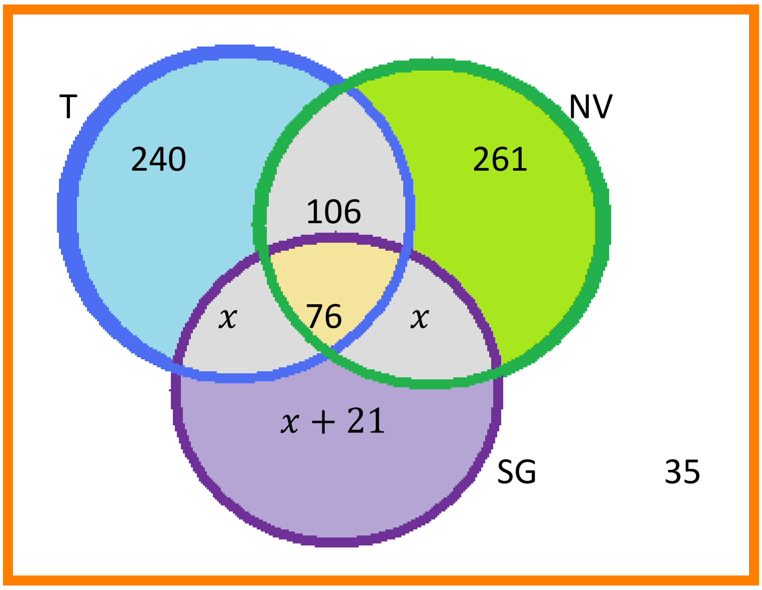
$= \frac{38}{75}$

f) P(MI4 and M3) = $\frac{45}{300} + \frac{21}{300}$

$= \frac{66}{300}$

$= \frac{11}{50}$

5. a)



b) $850 = 240 + 106 + 261 + x + 76 + x + x + 21 + 35$

$\therefore 850 = 739 + 3x$

$\therefore 3x = 111$

$\therefore x = 37$

c) P(all three) = $\frac{76}{850}$

$= \frac{38}{425}$

d) P(at least one) = $\frac{240}{850} + \frac{106}{850} + \frac{261}{850} + \frac{37}{850} + \frac{76}{850} + \frac{37}{850} + \frac{58}{850}$

$= \frac{815}{850}$

$= \frac{163}{170}$

e) P(at least two) = $\frac{37}{850} + \frac{106}{850} + \frac{76}{850} + \frac{37}{850}$

$$= \frac{256}{850} = \frac{128}{425}$$

f) P(no products) = $\frac{35}{850}$

$$= \frac{7}{170}$$

g) P(NV and T) = $\frac{106}{850} + \frac{76}{850}$

$$= \frac{182}{850}$$

$$= \frac{91}{425}$$

h) P(NV and SG) = $\frac{76}{850} + \frac{37}{850}$

$$= \frac{113}{850}$$

i) P(No NV) = $\frac{240}{850} + \frac{37}{850} + \frac{58}{850} + \frac{35}{850}$

$$= \frac{370}{850}$$

$$= \frac{37}{85}$$

<u>Activity 3</u>

a) Independent

b) Dependent

c) Dependent

d) Independent

<u>Activity 4</u>

1. a) P(top and didn't notice) = $\frac{15}{90}$

$$= \frac{1}{6}$$

b) P(bottom and didn't notice) = $\frac{25}{90}$

$$= \frac{5}{18}$$

c) P_E (top and noticed) = $\frac{45}{90} \times \frac{50}{90}$

$$= \frac{5}{18} \approx 0{,}2778$$

d) P_O (top and noticed) = $\frac{30}{90}$

$$= \frac{1}{3} \approx 0{,}33333..$$

e) Yes, there is a significant difference between the expected and observed values.

2. a) $a = 1000 - 135 - 58$ $b = 1000 - 258 - 58$

$\therefore a = 807$ $\therefore b = 684$

$c = 1000 + 1000$ OR = 1065 + 193 + 742

$\therefore c = 2000$

b) P(glue company and 6-7 hours) = $\frac{135}{2000}$

$= \frac{27}{400}$

c) P(competitor and 6-7 hours) = $\frac{58}{2000}$

$= \frac{29}{1000}$

d) P_E (4-5 hours and glue company) = $\frac{742}{2000} \times \frac{1000}{2000}$

$= \frac{371}{2000}$

e) P_O (4-5 hours and glue company) = $\frac{58}{2000}$

f) It is better → the experiment shows that the glue company's brand of glue is better.

3. P_E (Intensity above 50 and with chemical) = $\frac{18}{40} \times \frac{20}{40}$

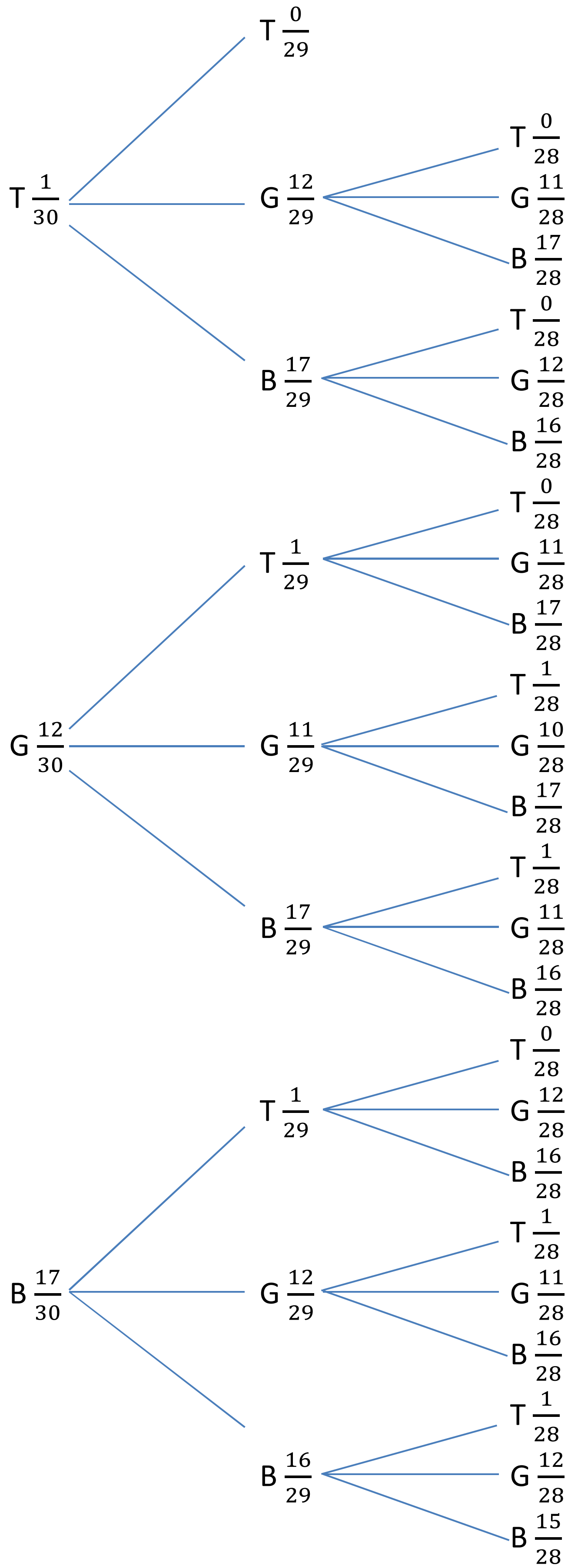
$= \frac{9}{40}$

P_O (Intensity above 50 and with chemical) = $\frac{15}{40}$

∴ Yes the experiment worked the observed value is much greater than the expected value.

Activity 5

1. a)



b) i) P(only boys) = $\frac{17}{30} \times \frac{16}{29} \times \frac{15}{28}$

$= \frac{34}{203}$

ii) P(at least one girl) → easier to calculate the probability of no girls winning a prize and then subtracting that answer from 1.

$\therefore = 1 - P(no\ girls)$

$\therefore = 1 - \left(\frac{1}{30} \times \frac{17}{29} \times \frac{16}{28} + \frac{17}{20} \times \frac{1}{29} \times \frac{16}{28} + \frac{17}{30} \times \frac{16}{29} \times \frac{1}{28} + \frac{17}{30} \times \frac{16}{29} \times \frac{15}{28}\right)$

$\therefore 1 - \left(\frac{34}{3045} + \frac{34}{3045} + \frac{34}{3045} + \frac{34}{203}\right)$

$\therefore = 1 - \frac{204}{1015}$

$\therefore = \frac{811}{1015}$

iii) P(teacher will win a prize) = $\frac{1}{30} \times \frac{12}{29} \times \frac{11}{28} + \frac{1}{30} \times \frac{12}{29} \times \frac{17}{28} + \frac{1}{30} \times \frac{17}{29} \times \frac{12}{28} + \frac{1}{30} \times \frac{17}{29} \times \frac{16}{28} + \frac{12}{30} \times \frac{1}{29} \times \frac{11}{28} + \frac{12}{30} \times \frac{1}{29} \times \frac{17}{28} + \frac{12}{20} \times \frac{11}{29} \times \frac{1}{28} + \frac{12}{30} \times \frac{17}{29} \times \frac{1}{28} + \frac{17}{30} \times \frac{1}{29} \times \frac{12}{28} + \frac{17}{30} \times \frac{1}{29} \times \frac{16}{28} + \frac{17}{30} \times \frac{12}{29} \times \frac{1}{28} + \frac{17}{30} \times \frac{16}{29} \times \frac{1}{28}$

$= \frac{11}{2030} + \frac{17}{2030} + \frac{17}{2030} + \frac{34}{3045} + \frac{11}{2030} + \frac{17}{2030} + \frac{11}{2030} + \frac{17}{2030} + \frac{17}{2030} + \frac{34}{3045} + \frac{17}{2030} + \frac{34}{3045}$

$= \frac{1}{10}$

iv) P(teacher will not win a prize) = 1 – P(teacher will win a prize)

$= 1 - \frac{1}{10}$

$= \frac{9}{10}$

v) P(2 girls and 1 boy) = $\frac{12}{30} \times \frac{11}{29} \times \frac{17}{28} + \frac{12}{30} \times \frac{17}{29} \times \frac{11}{28} + \frac{17}{30} \times \frac{12}{29} \times \frac{11}{28}$

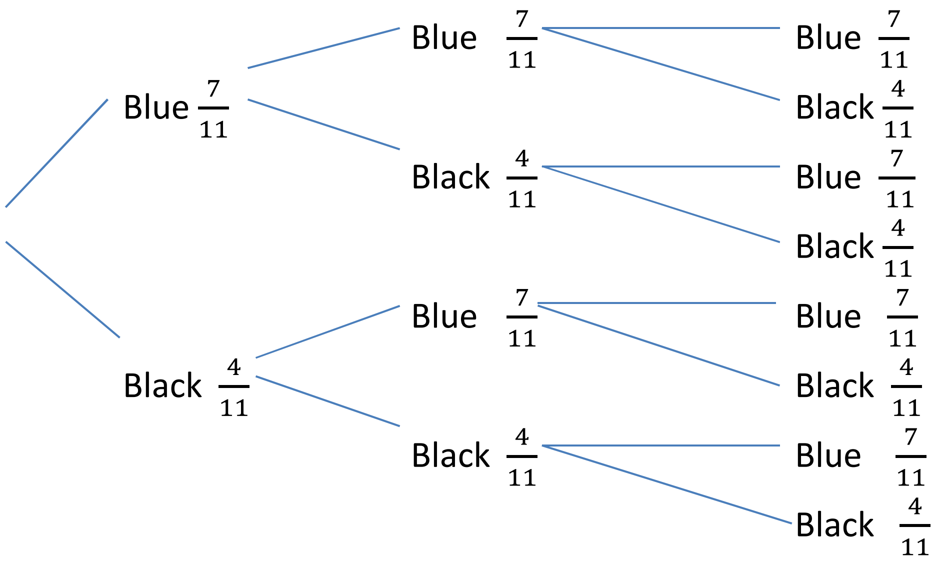
$= \frac{187}{2030} + \frac{187}{2030} + \frac{187}{2030}$

$= \frac{561}{2030}$

vi) P(1^{st} prize boy, 2^{nd} prize girl, 3^{rd} prize teacher) = $\frac{17}{30} \times \frac{12}{29} \times \frac{1}{28}$

$= \frac{17}{2030}$

2. a)



b) i) P(only black pens) = $\frac{4}{11} \times \frac{4}{11} \times \frac{4}{11}$

$= \frac{64}{1331}$

ii) P(at least one blue pen) → easier to work out the probability of no blue pens and then subtract the answer from 1.

= 1 – P(no blue pens) {ie only black pens}

$= 1 - \frac{64}{1331}$

$= \frac{1267}{1331}$

iii) P(2 blue pens and 1 black pen)

$= \frac{7}{11} \times \frac{7}{11} \times \frac{4}{11} + \frac{7}{11} \times \frac{4}{11} \times \frac{7}{11} + \frac{4}{11} \times \frac{7}{11} \times \frac{7}{11}$

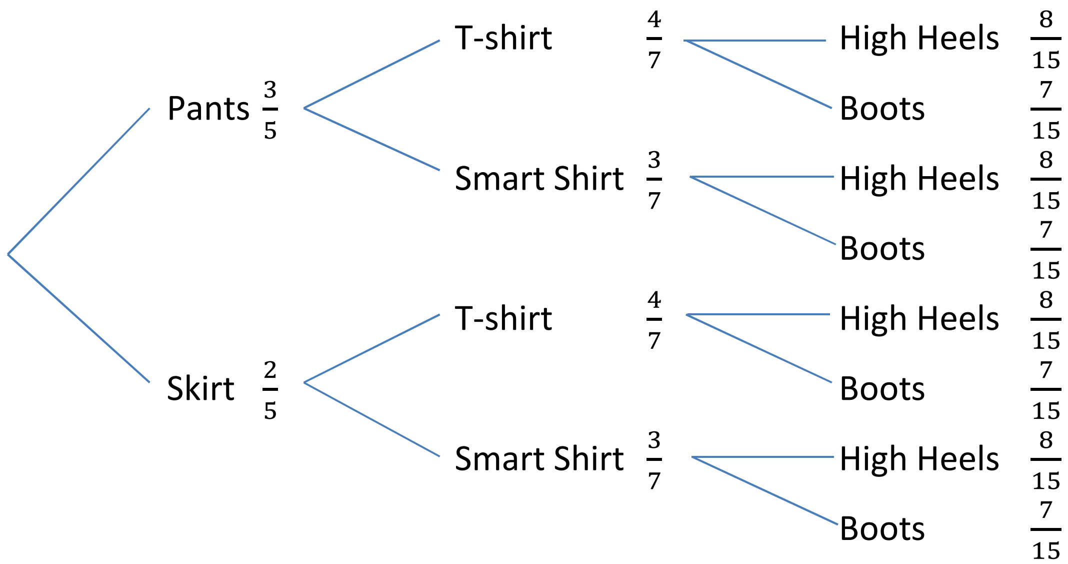
$= \frac{196}{1331} + \frac{196}{1331} + \frac{196}{1331}$

$= \frac{588}{1331}$

iv) P(black, blue, blue) = $\frac{4}{11} \times \frac{7}{11} \times \frac{7}{11}$

$= \frac{196}{1331}$

3.



a) P(pants; t-shirt; boots) = $\frac{3}{5} \times \frac{4}{7} \times \frac{7}{15}$

$= \frac{4}{25}$

b) P(skirt; smart shirt; high heels) = $\frac{2}{5} \times \frac{3}{7} \times \frac{8}{15}$

$= \frac{16}{175}$

c) P(pants; Smart shirt and heels OR t-shirt and heels)

$= \frac{3}{5} \times \frac{3}{7} \times \frac{8}{15} + \frac{3}{5} \times \frac{4}{7} \times \frac{8}{15}$

$= \frac{24}{175} + \frac{32}{175}$

$= \frac{8}{25}$

d) P(pants or skirt; Smart Shirt; Boots or high heels)

$= \frac{3}{5} \times \frac{3}{7} \times \frac{8}{15} + \frac{3}{5} \times \frac{3}{7} \times \frac{7}{15} + \frac{2}{5} \times \frac{3}{7} \times \frac{8}{15} \times \frac{2}{5} \times \frac{3}{7} \times \frac{7}{15}$

$= \frac{24}{175} + \frac{3}{25} + \frac{16}{175} + \frac{2}{25}$

$= \frac{3}{7}$ (i.e. just the probability of wearing smart shirt)

e) P(no skirt or smart shirt) = $\frac{3}{5} \times \frac{4}{7} \times \frac{8}{15} + \frac{3}{5} \times \frac{4}{7} \times \frac{7}{15}$

$= \frac{32}{175} + \frac{4}{12}$

$= \frac{12}{35}$

f) P(no pants or t-shirt) = $\frac{2}{5} \times \frac{3}{7} \times \frac{8}{15} + \frac{2}{5} \times \frac{3}{7} \times \frac{7}{15}$

$$= \frac{16}{175} + \frac{2}{25}$$

$$= \frac{6}{35}$$

4.

- a $\frac{1}{3}$
  - a $\frac{1}{3}$
    - a $\frac{1}{3}$
    - b $\frac{1}{3}$
    - c $\frac{1}{3}$
  - b $\frac{1}{3}$
    - a $\frac{1}{3}$
    - b $\frac{1}{3}$
    - c $\frac{1}{3}$
  - c $\frac{1}{3}$
    - a $\frac{1}{3}$
    - b $\frac{1}{3}$
    - c $\frac{1}{3}$ etc….

Therefore, P(of three in a row from the same seed packet) = $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

$$= \frac{1}{27}$$

<u>Activity 6</u>

1. $3 \times 2 = 6$
2. $3 \times 2 \times 4 = 24$
3. $6 \times 2 \times 3 \times 2 = 72$
4. $3 \times 4 = 12$
5. $6 \times 12 \times 4 \times 3 = 864\ combinations$

<u>Activity 7</u>

1. a) i) $\frac{(n+r-1)!}{r!(n-1)!}$

$$= \frac{(5+3-1)!}{3!(4)!}$$

$$= \frac{7!}{3!4!}$$

$$= 35$$

b) i) 5^3

$$= 125$$

ii) $\frac{n!}{r!(n-r)!}$

$= \frac{5!}{3!(5-3)!}$

$= \frac{5!}{3!2!}$

$= 10$

ii) $\frac{n!}{(n-r)!}$

$= \frac{5!}{(5-3)!}$

$= \frac{5!}{2!}$

$= 60$

2. $3 \times 10\ Cr\ 3$

$= 2 \times 120$

$= 360$

3. a) 11 possible toppings

b) $11\ Cr\ 4$

$= 330$

OR $\frac{n!}{r!(n-r)!}$

$= \frac{11!}{4!7!}$

$= 330$

4. $20\ \text{Pr}\ 3$

$= 6\ 840$

OR $\frac{n!}{(n-r)!}$

$= \frac{20!}{17!}$

$= 6\ 840$

5. $50\ Cr\ 5$

$= 2\ 118\ 760$

OR $\frac{n!}{r!(n-r)!}$

$= \frac{50!}{5!45!}$

$= 2\ 118\ 760$

6. $5\ \text{Pr}\ 3$

$= 60$

OR $\frac{n!}{(n-r)!}$

$= \frac{5!}{2!}$

$= 60$

P(guess right) = $\frac{1}{60}$

7. $15\ Cr\ 3$

$= 455$

OR $\frac{n!}{r!(n-r)!}$

$= \frac{15!}{3!12!}$

$= 455$

$\therefore$ The Chance of guessing the right combination is $\frac{1}{455}$ or 0,002