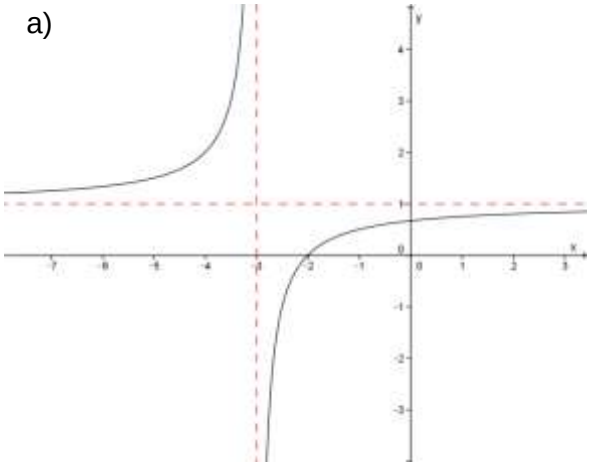
Worksheet 5: Functions

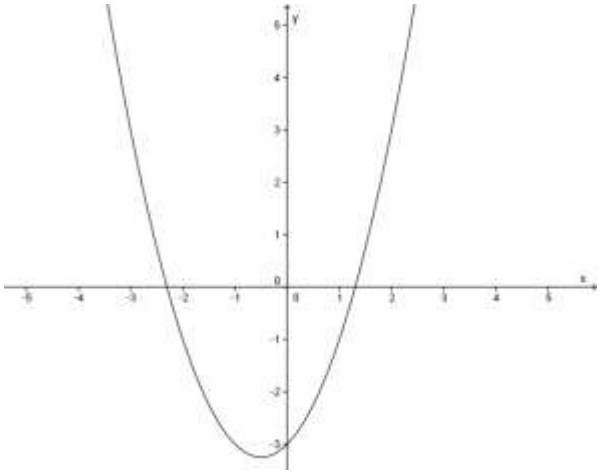
Grade 11 Mathematics

1. Identify the following functions as either a straight line, a parabola, a hyperbola or an exponential function and state whether the function is positive or negative: (K)

a)



b)



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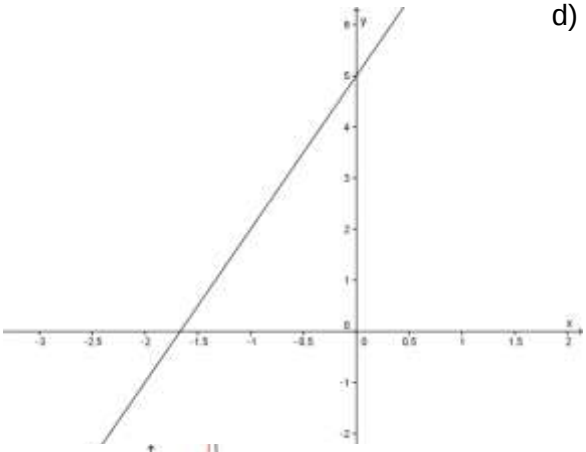
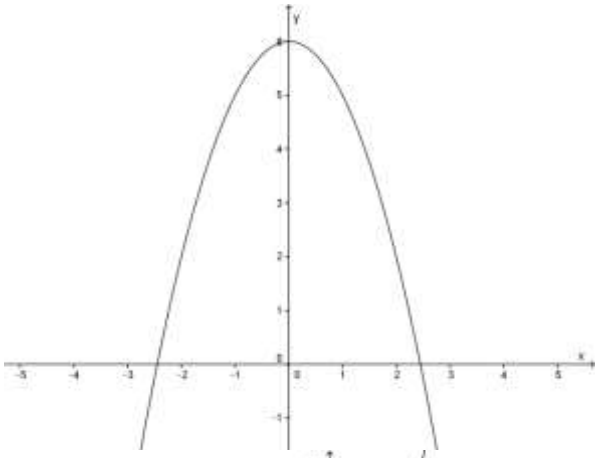
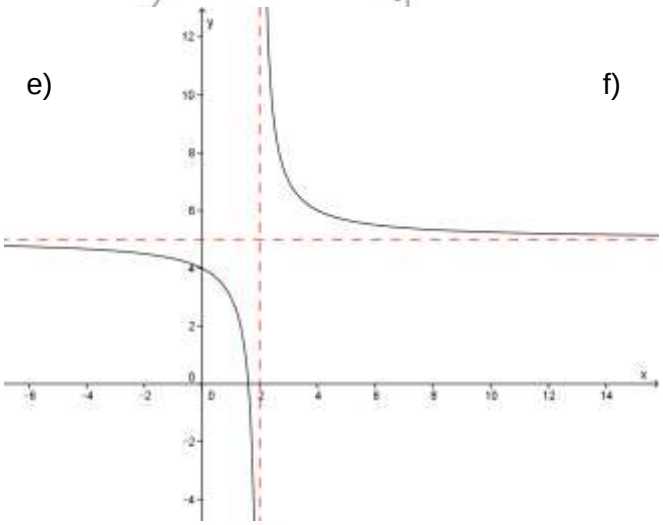


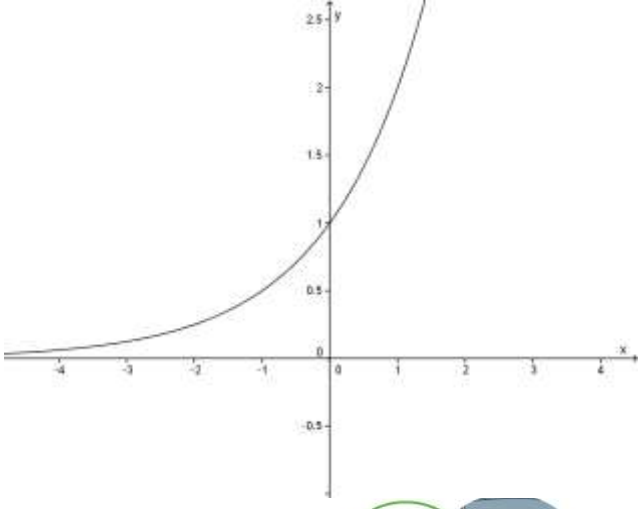
d)



e)



f)



2. Draw sketches of the following graphs labelling all intercepts: (R)

a) $y = -3^x$

b) $y = \frac{-8}{x-1} + 2$

c) $y = 2^x + 3$

d) $y = -\frac{1}{5}x - 5$

e) $y = \frac{3}{x-2} + 1$

f) $y = x^2 - 6x - 16$

g) $y = -x^2 + 1x + 6$

h) $y = 3^x - 1$

i) $y = \frac{1}{x+2} - 3$

j) $y = -x + 2$

3. Determine the equations of the following graphs:

a) A hyperbola ($y = \frac{k}{x-p} + q$) passing through the points (-4; -8.5) and (0; -7.5) with a y-asymptote of -8. (R)

b) A positive exponential graph ($y = a^{x+2} + q$) passing through the points (0; 2) and (-2; -1). (C)

c) A hyperbola ($y = \frac{-2}{x-p} + q$) passing through the points (-5; -3) and (-1; -5). (C)

d) A parabola ($y = ax^2 + bx + c$) passing through the points (-3; 22) and (2; 7) with a y-intercept of 7. (R)

e) A straight line perpendicular to $y = \frac{2}{3}x + 3$ and passing through the point (8; 0). (R)

f) A hyperbola ($y = \frac{6}{x-p} + q$) passing through the points (-5; -1) and (3; 3) with q > 0.(C)

g) An exponential graph ($y = 3^{x+p} + q$) passing through the points (2; 4) and (4; 12).(C)

h) A parabola ($y = -2x^2 + bx + c$) passing through the points (-2; -25) and (3; 0). (R)

i) A parabola ($y = 3x^2 + bx + c$) passing through the points (-9; 254) and (4; 7). (R)

j) An exponential graph ($y = -2^{x+p} + q$) passing through the points (1; 4) and (4; -3).(C)

4. Determine the $x-$ and y-intercepts of the graphs in question 3. (R)

5. Determine the new equation of $f(x) = 4x^2 + 5x + 1$ and $g(x) = \frac{2}{x+5} - 6$ if:

a) $f(x)$ is reflected about the x-axis. (R)

b) $g(x)$ is reflected about the x-axis. (R)

c) $f(x)$ is reflected about the y-axis. (R)

d) $g(x)$ is reflected about the y-axis. (R)

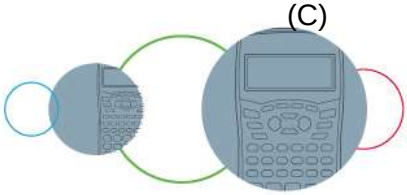
e) $f(x)$ is shifted 1 unit right and 6 units up. (C)

f) $g(x)$ is shifted 4 units left and 2 units up. (C)

g) $f(x)$ is shifted 9 units down. (R)

h) $g(x)$ is shifted 1 unit left and 2 units down. (C)

i) $g(x)$ is shifted 1 unit right and 4 units down. (C)



j) $f(x)$ is shifted 3 units right. (C)

6. Determine the points where the two graphs intersect:

a) $f(x) = -3x - 4$ and $g(x) = -x^2 + 2x - 3$ (C)

b) $h(x) = -x - 5$ and $j(x) = \frac{4}{x+6} + 8$ (C)

c) $k(x) = 3x + 12$ and $m(x) = x^2 - 3x - 4$ (R)

d) $n(x) = -x - 5$ and $p(x) = \frac{-1}{x+3} - 2$ (C)

e) $q(x) = -3x + 21$ and $r(x) = x^2 - 8x + 15$ (R)

f) $s(x) = 7x + 9$ and $t(x) = x^2 + 2x - 15$ (R)

g) $v(x) = -5x + 3$ and $w(x) = \frac{-4}{x+3} - 1$ (C)

h) $y(x) = 3x - 8\frac{1}{2}$ and $z(x) = \frac{3}{x-4} + 8$ (C)

i) $b(x) = -4x$ and $c(x) = \frac{-1}{x+2} - 3$ (C)

j) $d(x) = \frac{1}{2}x + 7$ and $f(x) = 4x^2 - 6x + 4$ (C)

7. Determine the equations of the axes of symmetry for the following graphs: (R)

a) $y = -x^2 + 2x - 3$

b) $y = \frac{4}{x+6} + 8$

c) $y = x^2 - 3x - 4$

d) $y = \frac{-1}{x+3} - 2$

e) $y = x^2 - 8x + 15$

f) $y = 3^x$

g) $y = \frac{2}{x-1} + 4$

h) $y = x + 2$

i) $y = -2x^2 + 7x - 9$

j) $y = \frac{1}{x-1} + 3$

8. Determine the turning points of the following parabolas and say whether the point is at a maximum or minimum: (C)

a) $y = -x^2 + 2x - 3$

b) $y = x^2 - 3x - 4$

c) $y = x^2 - 8x + 15$

d) $y = -2x^2 + 7x - 9$

e) $y = x^2 + 8x + 9$

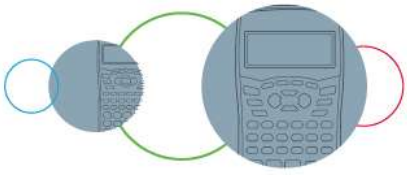
f) $y = 2x^2 + 5x - 6$

g) $y = -3x^2 + 10x - 9$

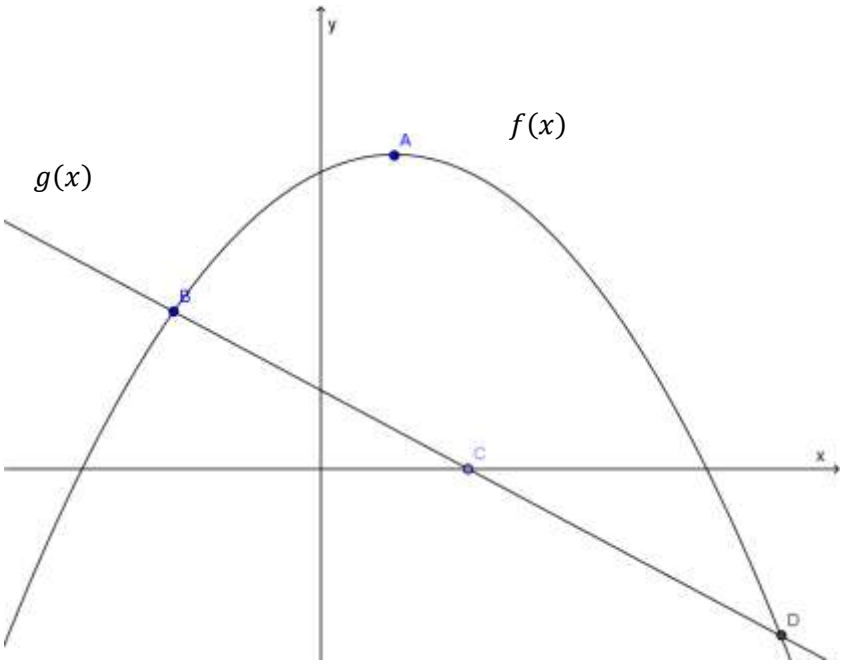
h) $y = 5x^2 - 9x + 4$

i) $y = -\frac{1}{2}x^2 - 3x - 2$

j) $y = \frac{1}{4}x^2 + \frac{1}{2}x + 1$



9. Given the graphs $f(x) = ax^2 + bx + c$ and $g(x) = mx + d$ below. $f(x)$ has a turning point at A(1; 2) and intersects with $g(x)$ at the point B(-2; 1).



a) Determine the values of a, b, and c. (R)

b) Given that $g(x)$ is perpendicular to $y = 4x + 2$, determine the values of m and c. (R)

c) Determine point D, the other point of intersection of f and g. (C)

d) Determine the x- and y-intercepts for graphs f and g. (R)

e) Give the values for x where $f(x) \geq g(x)$ (P)

f) Determine the distance between f and g when $x = 0$. (R)

g) Determine the value of x for which the distance between f and g is at a maximum. (P)

h) Give the equation of the axis of symmetry for $f(x)$. (C)

i) Give the equation of $m(x)$ which is the reflection of $f(x)$ about the y-axis. (R)

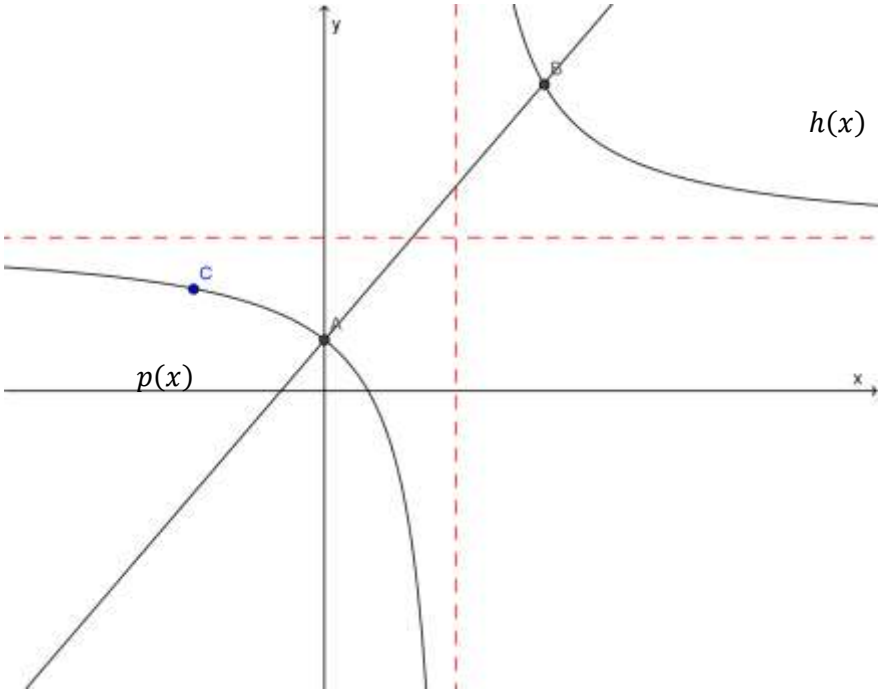
j) Determine the points of intersection of $m(x)$ and $g(x)$. (C)

k) Determine the x- and y-intercepts and turning point of $m(x)$. (C)

l) Compare the values found in question (k) with those found in (d). What do you notice? (P)

m) Determine the values for x where $m(x) = f(x)$. (C)

10. Given the graphs $h(x) = \frac{k}{x-p} + q$ and $p(x) = mx + c$ below, with point A and B as the points of intersection between the two graphs. C (-1; 2) is a point on the graph $h(x)$.



a) The asymptotes for the hyperbola are $x = 1$ and $y = 3$, determine the equations of $h(x)$. (R)

b) Determine the coordinates of A, the y-intercept of $h(x)$. (R)

c) Hence, or otherwise determine the equation of $p(x)$ given that $p(x)$ passes through the point $\left(\frac{1}{2}; 3\right)$. (R)

d) Determine B, the other point of intersection. (C)

e) Determine the equations for the axes of symmetry for $h(x)$. (C)

f) For which values of x does the graph of $h(x)$ not exist? (P)

g) Determine the x- and y-intercepts of both $h(x)$ and $p(x)$. (C)

h) For which values of x is $h(x) \leq p(x)$? (P)

i) Determine the height of $h(x)$ above $p(x)$ at the point C. (P)