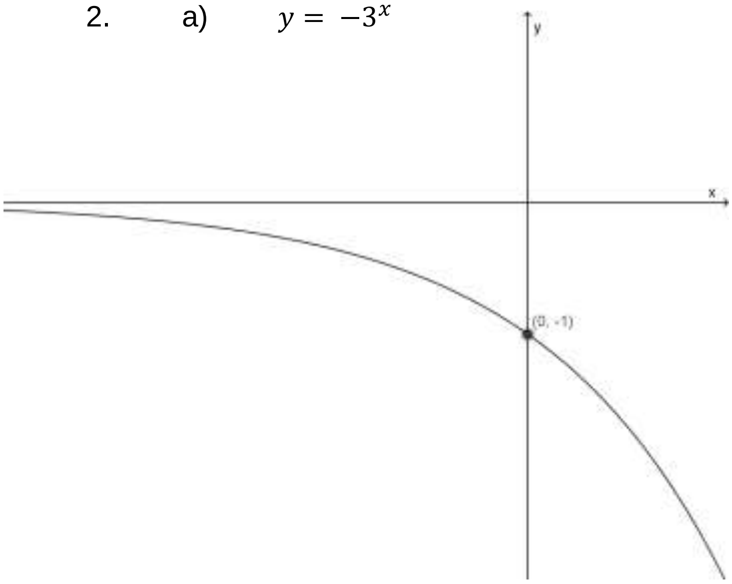
SHARP

Worksheet 5 Memo: Functions

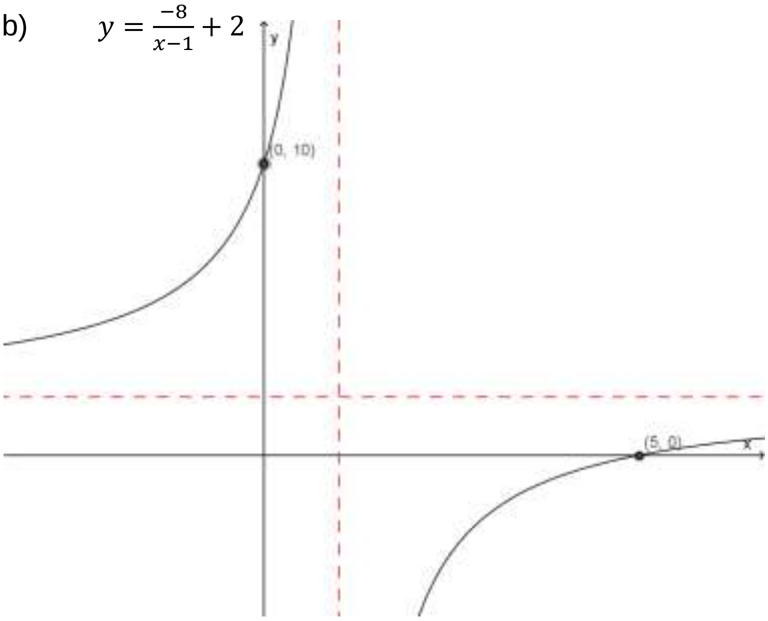
Grade 11 Mathematics

1. a) hyperbola – negative
   b) parabola – positive
   c) straight-line – positive
   d) parabola – negative
   e) hyperbola – positive
   f) exponential – positive

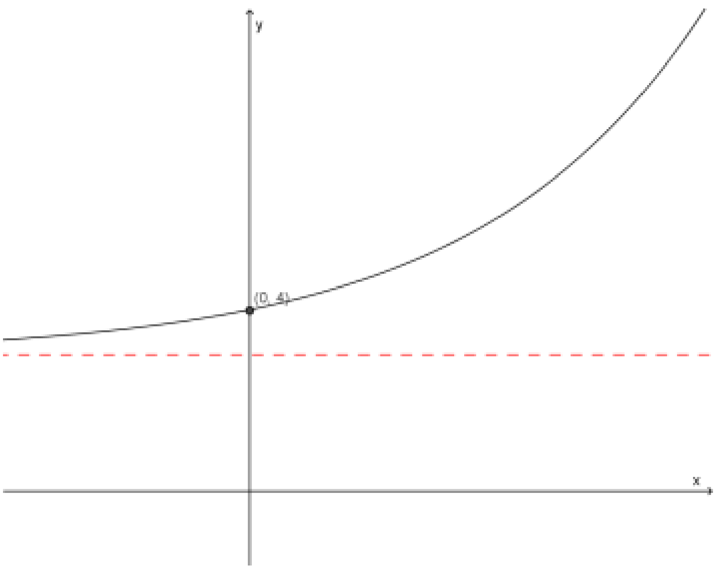
2. a) $y = -3^x$



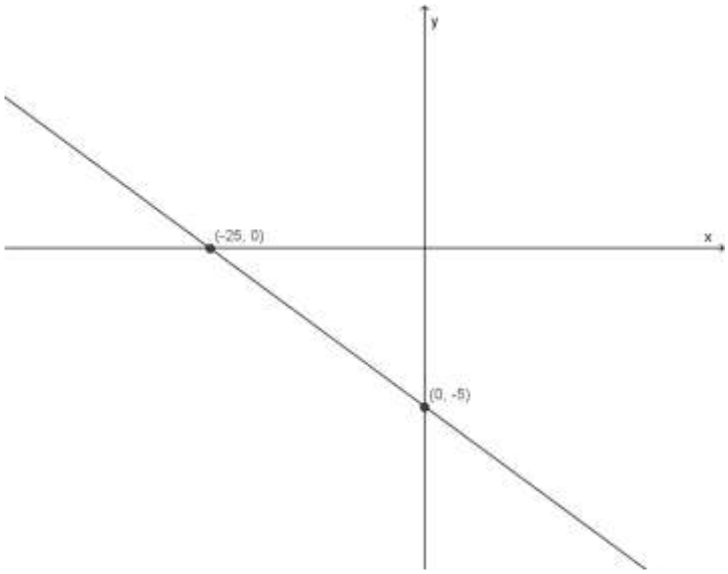
b) $y = \frac{-8}{x-1} + 2$



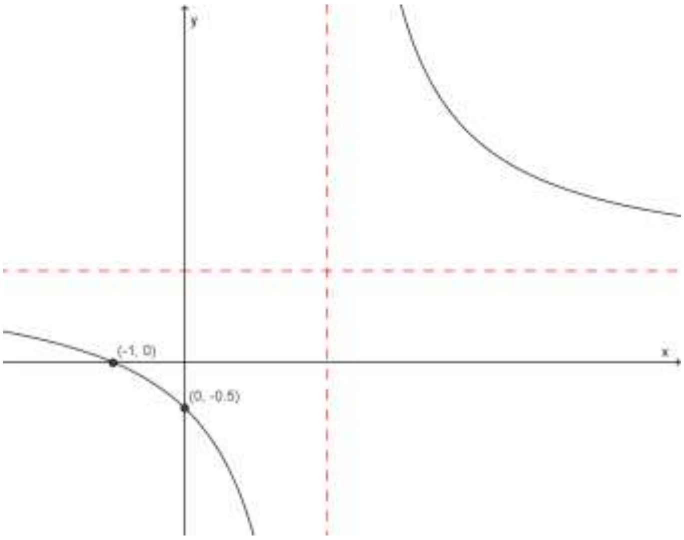
c) $y = 2^x + 3$



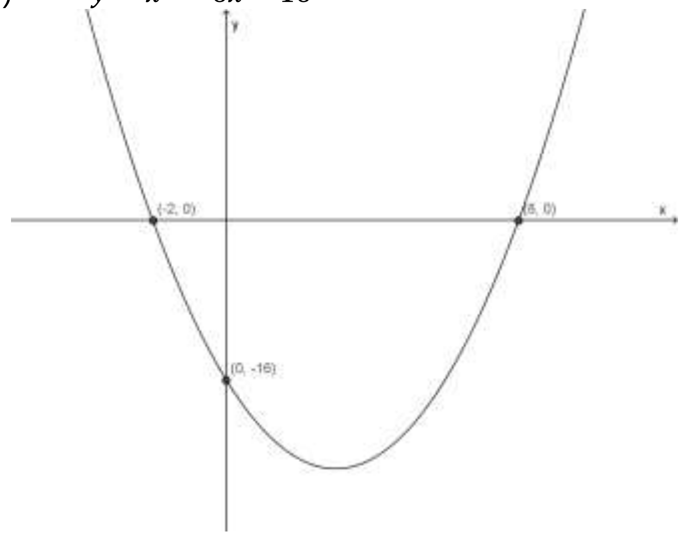
d) $y = -\frac{1}{5}x - 5$



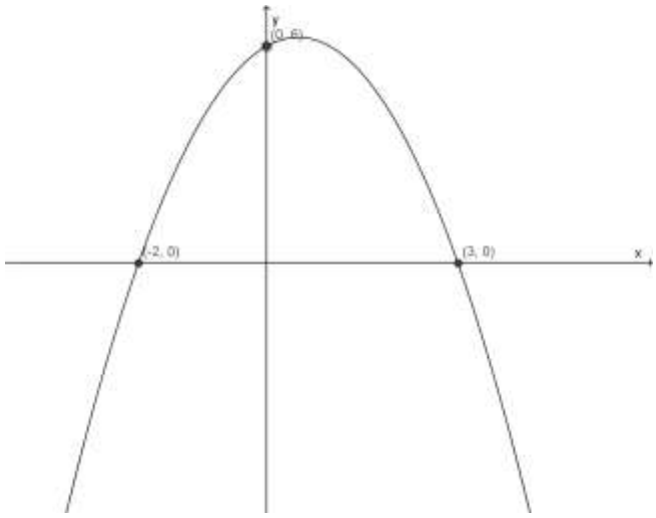
e) $y = \frac{3}{x-2} + 1$



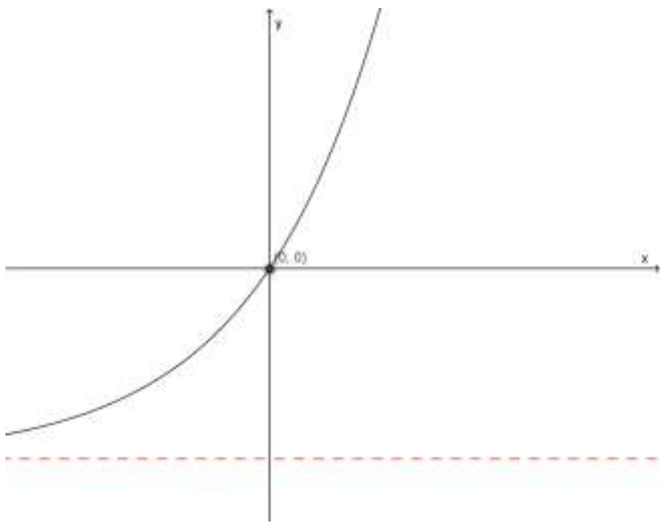
f) $y = x^2 - 6x - 16$



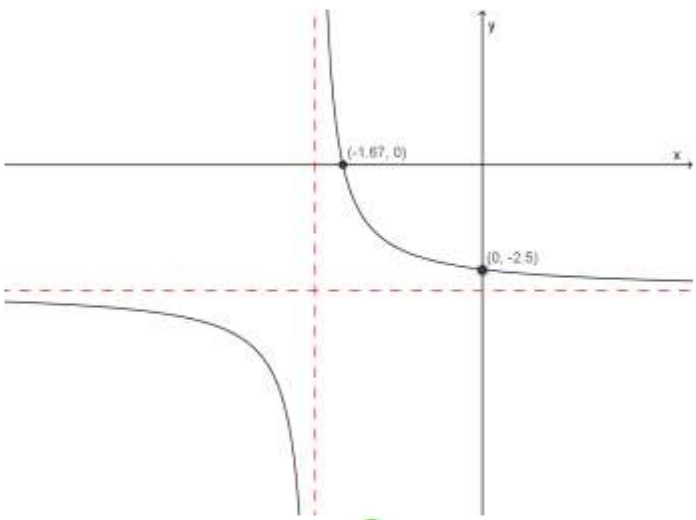
g) $y = -x^2 + 1x + 6$



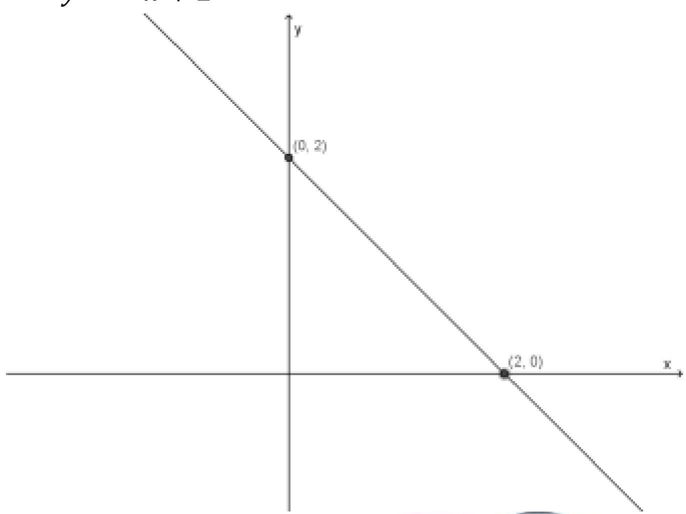
h) $y = 3^x - 1$



i) $y = \frac{1}{x+2} - 3$



j) $y = -x + 2$



3. a) $y = \frac{k}{x-p} + q$ (-4; -8.5) and (0; -7.5) and y =-8

$\therefore y = \frac{k}{x-p} - 8$

$\therefore\ -8\frac{1}{2} = \frac{k}{-4-p} - 8$ and $-7\frac{1}{2} = \frac{k}{0-p} - 8$

$\therefore\ -\frac{1}{2} = \frac{k}{-4-p}$ $\frac{1}{2} = \frac{k}{-p}$

$\therefore 2 + \frac{1}{2}p = k \ldots$ 1 $-\frac{1}{2}p = k\ \ldots$ 2

Subs 1 into 2:

$\therefore 2 + \frac{1}{2}p =\ -\frac{1}{2}p$

$\therefore p =\ -2$ Subs back into 2:

$\therefore k =\ -\frac{1}{2}(-2)$

$\therefore k = 1$ $\therefore y = \frac{1}{x+2} - 8$

b) $y = a^{x+2} + q$ (0; 2) and (-2; -1)

$\therefore\ -1 = a^{-2+2} + q$

$\therefore\ -1 = a^0 + q$ and $\therefore 2 = a^{0+2} - 2$

$\therefore\ -1 = 1 + q$ $\therefore 4 = a^2$

$\therefore q =\ -2$ $\therefore a = 2$ a cannot be negative.

$\therefore y = 2^{x+2} - 2$

c) $y = \frac{-2}{x-p} + q$ (-5; -3) and (-1; -5)

$\therefore\ -3 = \frac{-2}{-5-p} + q$ and $-5 = \frac{-2}{-1-p} + q$

$\therefore\ -3 - q = \frac{-2}{-5-p}$ $-5 - q = \frac{-2}{-1-p}$

$\therefore (-3-q)(-5-p) =\ -2$ $(-5-q)(-1-p) =\ -2$

$\therefore 15 + 3p + 5q + qp =\ -2$ $5 + 5p + q + qp =\ -2\ \ldots.$ 2

$\therefore 3p + pq =\ -17 - 5q$

$\therefore p(3+q) =\ -17 - 5q$

$\therefore p = \frac{-17-5q}{3+q} \ldots$ 1 Subs 1 into 2:

$\therefore 5 + 5\left(\frac{-17-5q}{3+q}\right) + q + q\left(\frac{-17-5q}{3+q}\right) =\ -2$

$\therefore \frac{-85-25q}{3+q} + q + \frac{-17q-5q^2}{3+q} =\ -7$

$\therefore\ -85 - 25q + q(3+q) - 17q - 5q^2 =\ -7(3+q)$

$\therefore\ -85 - 42q - 5q^2 + 3q + q^2 + 21 + 7q = 0$

$\therefore\ -4q^2 - 32q - 64 = 0$

$\therefore q^2 + 8q + 16 = 0$

$\therefore (q+4)(q+4)=0$ $\therefore q = -4$

Subs back into 1:

$\therefore p = \frac{-17-5(-4)}{3-4} = -3$ $\therefore y = \frac{-2}{x+3} - 4$

d) $y = ax^2 + bx + c$ (-3; 22) and (2; 7) with c = 7.

$\therefore y = ax^2 + bx + 7$

$\therefore 22 = a(-3)^2 + b(-3) + 7$ and $7 = a(2)^2 + b(2) + 7$

$\therefore 15 = 9a - 3b$ $0 = 4a + 2b$ … 2

$\therefore 15 - 9a = -3b$

$\therefore 3a - 5 = b$ … 1

Subs 1 into 2:

$\therefore 0 = 4a + 2(3a-5)$

$\therefore 0 = 4a + 6a - 10$

$\therefore 10a = 10$

$\therefore a = 1$

Subs back into 1:

$\therefore 3(1) - 5 = b$

$\therefore b = -2$ $\therefore y = x^2 - 2x + 7$

e) $y = mx + c$ $m_1 = \frac{2}{3} \rightarrow$ $\therefore m_2 \times \frac{2}{3} = -1$

$\therefore m_2 = -\frac{3}{2}$

$\therefore y = -\frac{3}{2}x + c$ Subs (8; 0)

$\therefore 0 = -\frac{3}{2}(8) + c$

$\therefore c = 12$ $\therefore y = -\frac{3}{2}x + 12$

f) $y = \frac{6}{x-p} + q$ (-5; -1) and (3; 3)

$\therefore -1 = \frac{6}{-5-p} + q$ and $3 = \frac{6}{3-p} + q$

$\therefore -1 - q = \frac{6}{-5-p}$ $3 - q = \frac{6}{3-p}$

$\therefore (-1-q)(-5-p) = 6$ $(3-q)(3-p) = 6$

$\therefore 5 + p + 5q + qp = 6$ $9 - 3p - 3q + qp - 6 = 0$

$\therefore p + qp = 1 - 5q$ $3 - 3p - 3q + qp = 0$ …2

$\therefore p(1+q) = 1 - 5q$

$\therefore p = \frac{1-5q}{1+q}$ … 1

Subs 1 into 2:

$\therefore 3 - 3\left(\frac{1-5q}{1+q}\right) - 3q + q\left(\frac{1-5q}{1+q}\right) = 0$

$\therefore 3 - \frac{3-15q}{1+q} - 3q + \frac{q-5q^2}{1+q} = 0$

$\therefore 3(1+q) - 3 + 15q - 3q(1+q) + q - 5q^2 = 0$

$\therefore 3 + 3q - 3 + 15q - 3q - 3q^2 + q - 5q^2 = 0$

$\therefore \ -8q^2 + 16q = 0$

$\therefore -8q(q-2) = 0$

$\therefore q = 0 \quad or\ q = 2$

$\therefore p = \frac{1-5(2)}{1+2}$ N/A

$\therefore p = \ -3$ $\therefore y = \frac{6}{x+3} + 2$

g) $y = 3^{x+p} + q$ (2; 4) and (4; 12)

$\therefore 4 = 3^{2+p} + q$ and $12 = 3^{4+p} + q$

$\therefore 4 - q = 3^2.3^p$ $12 - q = 3^4 3^p$

$\therefore \frac{4-q}{9} = 3^p \ \ldots$ 1 $\frac{12-q}{81} = 3^p \ \ldots$ 2

Subs 1 into 2:

$\therefore \frac{4-q}{9} = \frac{12-q}{81}$

$\therefore 9(4-q) = 12 - q$

$\therefore 36 - 9q = 12 - q$

$\therefore \ -8q = \ -24$

$\therefore q = 3$ Subs back into 1:

$\therefore 3^p = \frac{4-3}{9}$

$\therefore 3^p = \frac{1}{9} = 3^{-2}$

$\therefore p = \ -2$ $\therefore y = 3^{x-2} + 3$

h) $y = -2x^2 + bx + c$ (-2; -25) and (3; 0)

$\therefore \ -25 = \ -2(-2)^2 + b(-2) + c$ and $0 = \ -2(3)^2 + b(3) + c$

$\therefore \ -25 = \ -8 - 2b + c$ $0 = \ -18 + 3b + c$

$\therefore \ -17 = \ -2b + c$ $c = 18 - 3b \ \ldots$ 2

$\therefore 2b - 17 = c \ \ldots$ 1 Subs 1 into 2:

$\therefore 18 - 3b = 2b - 17$

$\therefore 5b = 35$

$\therefore b = 7$Subs back into 1:

$\therefore c = 2(7) - 17$

$\therefore c = \ -3$ $\therefore y = \ -2x^2 + 7x - 3$

i) $y = 3x^2 + bx + c$ (-9; 254) and (4; 7).

$\therefore 254 = 3(-9)^2 + b(-9) + c$ and $7 = 3(4)^2 + b(4) + c$

$\therefore 254 = 243 - 9b + c$ $7 = 48 + 4b + c$

$\therefore 9b + 11 = c$… 1 $-41 - 4b = c$ …2

Subs 1 into 2:

$\therefore 9b + 11 = -41 - 4b$

$\therefore 13b = -52$

$\therefore b = -4$ Subs back into 1:

$\therefore c = 9(-4) + 11$

$\therefore c = -25$

$\therefore y = 3x^2 - 4x - 25$

j) $y = -2^{x+p} + q$ (1; 4) and (4; -3)

$\therefore 4 = -2^{1+p} + q$ and $-3 = -2^{4+p} + q$

$\therefore 4 - q = -2.2^p$ $-3 - q = -2^4.2^p$

$\therefore \frac{4-q}{-2} = 2^p$… 1 $\frac{-3-q}{-2^4} = 2^p$ … 2

Subs 1 into 2:

$\therefore \frac{4-q}{-2} = \frac{-3-q}{-2^4}$

$\therefore 8(4 - q) = -3 - q$

$\therefore 32 - 8q = -3 - q$

$\therefore -7q = -35$

$\therefore q = 5$ Subs back into 1:

$\therefore 2^p = \frac{4-5}{-2}$

$\therefore 2^p = \frac{1}{2} = 2^{-1}$

$\therefore p = -1$ $\therefore y = -2^{x-1} + 5$

4. a) $y = \frac{1}{x+2} - 8$

$\therefore$ Let $y = 0$ Let $x = 0$

$\therefore 0 = \frac{1}{x+2} - 8$ $y = \frac{1}{0+2} - 8$

$\therefore 8 = \frac{1}{x+2}$ $y = \frac{1}{2} - 8$

$\therefore 8x + 16 = 1$ $y = -7\frac{1}{2}$

$\therefore 8x = -15$

$\therefore x = -\frac{15}{8} = -1\frac{7}{8}$ $\therefore \left(-1\frac{7}{8}; 0\right)$ *and* $\left(0; -7\frac{1}{2}\right)$

b) $y = 2^{x+2} - 2$

$\therefore$ Let $y = 0$

$\therefore 0 = 2^{x+2} - 2$

$\therefore 2 = 2^{x+2}$

$\therefore x + 2 = 1$

$\therefore x = -1$

Let $x = 0$

$y = 2^{0+2} - 2$

$y = 4 - 2$

$y = 2$

$\therefore (-1; 0)\ and\ (0; 2)$

c) $y = \frac{-2}{x+3} - 4$

$\therefore$ Let $y = 0$

$\therefore 0 = \frac{-2}{x+3} - 4$

$\therefore 4 = -\frac{2}{x+3}$

$\therefore 4x + 12 = -2$

$\therefore 4x = -14$

$\therefore x = -3\frac{1}{2}$

Let $x = 0$

$y = \frac{-2}{0+3} - 4$

$y = -\frac{2}{3} - 4$

$y = -4\frac{2}{3}$

$\therefore \left(-3\frac{1}{2}; 0\right)\ and\ \left(0; -4\frac{2}{3}\right)$

d) $\therefore y = x^2 - 2x + 7$

$\therefore$ Let $y = 0$

$\therefore 0 = x^2 - 2x + 7$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(7)}}{2(1)}$

$\therefore x = does\ not\ exist$

Let $x = 0$

$y = (0)^2 - 2(0) + 7$

$y = 7$

$\therefore (0; 7)$ and no x-intercepts

e) $y = -\frac{3}{2}x + 12$

$\therefore$ Let $y = 0$

$\therefore 0 = -\frac{3}{2}x + 12$

$\therefore \frac{3}{2}x = 12$

$\therefore x = 8$

Let $x = 0$

$y = -\frac{3}{2}(0) + 12$

$y = 12$

$\therefore (8; 0)\ and\ (0; 12)$

f) $y = \frac{6}{x+3} + 2$

$\therefore$ Let $y = 0$

$\therefore 0 = \frac{6}{x+3} + 2$

$\therefore -2 = \frac{6}{x+3}$

$\therefore -2x - 6 = 6$

$\therefore -2x = 12$

$\therefore x = -6$

Let $x = 0$

$y = \frac{6}{0+3} + 2$

$y = \frac{6}{3} + 2$

$y = 2 + 2 = 4$

$\therefore (-6; 0)\ and\ (0; 4)$

g) $y = 3^{x-2} + 3$

$\therefore$ Let $y = 0$

$\therefore 0 = 3^{x-2} + 3$

$\therefore\ -3 = 3^{x-2}$

$\therefore x\ does\ not\ exist$

$y = 0$ is an asymptote

Let $x = 0$

$y = 3^{0-2} + 3$

$y = 3^{-2} + 3$

$y = 3\frac{1}{9}$

$\therefore \left(0; 3\frac{1}{9}\right)$

h) $y = -2x^2 + 7x - 3$

$\therefore$ Let $y = 0$

$\therefore 0 = -2x^2 + 7x - 3$

$\therefore 0 = 2x^2 - 7x + 3$

$\therefore 0 = (2x - 1)(x - 3)$

$x = \frac{1}{2}\quad or\ x = 3$

Let $x = 0$

$y = -2(0)^2 + 7(0) - 3$

$y = -3$

$\therefore \left(\frac{1}{2}; 0\right), (3; 0)\ and\ (0;\ -3)$

i) $y = 3x^2 - 4x - 25$

$\therefore$ Let $y = 0$

$\therefore 0 = 3x^2 - 4x - 25$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-25)}}{2(3)}$

$\therefore x = 3{,}63 \quad or \quad x = -2{,}30$

Let $x = 0$

$y = 3(0)^2 - 4(0) - 25$

$y = -25$

$\therefore (3{,}63; 0), (-2{,}30; 0)\ and\ (0;\ -25)$

j) $y = -2^{x-1} + 5$

$\therefore$ Let $y = 0$

$\therefore 0 = -2^{x-1} + 5$

$\therefore\ 2^{x-1} = 5$

$\therefore x - 1 = 2{,}32$ (trial and error)

$\therefore\ x = 3{,}32$

Let $x = 0$

$y = -2^{0-1} + 5$

$y = -\frac{1}{2} + 5$

$y = 4\frac{1}{2}$

$\therefore (3{,}32; 0)\ and\ \left(0; 4\frac{1}{2}\right)$

5. a) $f(x) = 4x^2 + 5x + 1$

$\therefore -y = 4x^2 + 5x + 1$

$\therefore y = -4x^2 - 5x - 1$

b) $g(x) = \frac{2}{x+5} - 6$

$\therefore -y = \frac{2}{x+5} - 6$

$\therefore y = -\frac{2}{x+5} + 6$

c) $f(x) = 4x^2 + 5x + 1$

$\therefore y = 4(-x)^2 + 5(-x) + 1$

$\therefore y = 4x^2 - 5x + 1$

d) $g(x) = \frac{2}{x+5} - 6$

$\therefore y = \frac{2}{-x+5} - 6$

$\therefore y = \frac{-2}{x-5} - 6$

e) $f(x) = 4x^2 + 5x + 1$

$\therefore f(x-1) + 6 = 4(x-1)^2 + 5(x-1) + 1 + 6$

$\therefore y = 4(x^2 - 2x + 1) + 5x - 5 + 7$

$\therefore y = 4x^2 - 8x + 4 + 5x + 2$

$\therefore y = 4x^2 - 3x + 6$

f) $g(x) = \frac{2}{x+5} - 6$

$\therefore g(x+4) + 2 = \frac{2}{x+4+5} - 6 + 2$

$\therefore y = \frac{2}{x+9} - 4$

g) $f(x) = 4x^2 + 5x + 1$

$\therefore f(x) - 9 = 4x^2 + 5x + 1 - 9$

$\therefore y = 4x^2 + 5x - 8$

h) $g(x) = \frac{2}{x+5} - 6$

$\therefore g(x+1) - 2 = \frac{2}{x+1+5} - 6 - 2$

$\therefore y = \frac{2}{x+6} - 8$

i) $g(x) = \frac{2}{x+5} - 6$

$\therefore g(x-1) - 4 = \frac{2}{x-1+5} - 6 - 4$

$\therefore y = \frac{2}{x+4} - 10$

j) $f(x) = 4x^2 + 5x + 1$

$\therefore f(x-3) = 4(x-3)^2 + 5(x-3) + 1$

$\therefore y = 4(x^2 - 6x + 9) + 5x - 15 + 1$

$\therefore y = 4x^2 - 24x + 36 + 5x - 14$

$\therefore y = 4x^2 - 19x + 22$

6. a) $f(x) = -3x - 4$ and $g(x) = -x^2 + 2x - 3$

$f(x) = g(x)$

$\therefore -3x - 4 = -x^2 + 2x - 3$

$\therefore x^2 - 5x - 1 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-1)}}{2(1)}$

$\therefore x = 5{,}19 \quad or \quad x = -0.19$

Subs back into $f(x)$ for y:

$\therefore y = -3(5{,}19) - 4$ OR $y = -3(-0.19) - 4$

$\therefore y = -19{,}57$ $\qquad y = -3{,}43$

$\therefore (5{,}19;\ -19{,}57) \quad and \quad (-0.19;\ -3{,}43)$

b) $h(x) = -x - 5$ and $j(x) = \frac{4}{x+6} + 8$

$h(x) = j(x)$

$\therefore -x - 5 = \frac{4}{x+6} + 8$

$\therefore -x - 13 = \frac{4}{x+6}$

$\therefore (-x - 13)(x + 6) = 4$

$\therefore -x^2 - 6x - 13x - 78 - 4 = 0$

$\therefore x^2 + 19x + 82 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-19 \pm \sqrt{(19)^2 - 4(1)(82)}}{2(1)}$

$\therefore x = -6{,}63 \quad or \quad x = -12{,}37$ Subs back into $h(x)$ for y:

$\therefore y = -(-6{,}63) - 5$ OR $y = -(-12{,}37) - 5$

$\therefore y = 1{,}63$ $y = 7{,}37$

$\therefore (-6{,}63; 1{,}63)\ and\ (-12{,}37; 7{,}37)$

c) $k(x) = 3x + 12$ and $m(x) = x^2 - 3x - 4$

$k(x) = m(x)$

$\therefore 3x + 12 = x^2 - 3x - 4$

$\therefore 0 = x^2 - 6x - 16$

$\therefore 0 = (x - 8)(x + 2)$

$\therefore x = 8 \quad or \quad x = -2$ Subs back into $k(x)$ for y:

$\therefore y = 3(8) + 12$ OR $y = 3(-2) + 12$

$\therefore y = 36$ $y = 6$

$\therefore (8; 36)\ and\ (-2; 6)$

d) $n(x) = -x - 5$ and $p(x) = \frac{-1}{x+3} - 2$

$n(x) = p(x)$

$\therefore -x - 5 = \frac{-1}{x+3} - 2$

$\therefore -x - 3 = \frac{-1}{x+3}$

$\therefore (-x - 3)(x + 3) = -1$

$\therefore -x^2 - 3x - 3x - 9 + 1 = 0$

$\therefore x^2 + 6x + 8 = 0$

$\therefore (x + 4)(x + 2) = 0$

$\therefore x = -4 \quad or \quad x = -2$ Subs back into $n(x)$ for y:

$\therefore y = -(-4) - 5$ OR $y = -(-2) - 5$

$\therefore y = -1$ $y = -3$

$\therefore (-4;\ -1)\ and\ (-2;\ -3)$

e) $q(x) = -3x + 21$ and $r(x) = x^2 - 8x + 15$

$q(x) = r(x)$

$\therefore -3x + 21 = x^2 - 8x + 15$

$\therefore 0 = x^2 - 5x - 6$

$\therefore 0 = (x - 6)(x + 1)$

$\therefore x = 6 \ \ or \ \ x = -1$ Subs back into $q(x)$ for y:

$\therefore y = -3(6) + 21$ OR $y = -3(-1) + 21$

$\therefore y = 3$ $y = 24$

$\therefore (6; 3) \ and \ (-1; 24)$

f) $s(x) = 7x + 9$ and $t(x) = x^2 + 2x - 15$

$s(x) = t(x)$

$\therefore 7x + 9 = x^2 + 2x - 15$

$\therefore 0 = x^2 - 5x - 24$

$\therefore 0 = (x - 8)(x + 3)$

$\therefore x = 8 \ \ or \ \ x = -3$ Subs back into $s(x)$ for y:

$\therefore y = 7(8) + 9$ OR $y = 7(-3) + 9$

$\therefore y = 65$ $y = -12$

$\therefore (8; 65) \ and \ (-3; \ -12)$

g) $v(x) = -5x + 3$ and $w(x) = \frac{-4}{x+3} - 1$

$v(x) = w(x)$

$\therefore -5x + 3 = \frac{-4}{x+3} - 1$

$\therefore -5x + 4 = \frac{-4}{x+3}$

$\therefore (-5x + 4)(x + 3) = -4$

$\therefore -5x^2 - 15x + 4x + 12 + 4 = 0$

$\therefore 0 = 5x^2 + 11x - 16$

$\therefore 0 = (5x + 16)(x - 1)$

$\therefore x = -\frac{16}{5} \ \ or \ \ x = 1$ Subs back into $v(x)$ for y:

$\therefore y = -5\left(-\frac{16}{5}\right) + 3$ OR $y = -5(1) + 3$

$\therefore y = 16 + 3$ $y = -2$

$\therefore y = 19$

$\therefore \left(-3\frac{1}{5}; 19\right) \ and \ (1; \ -2)$

h) $y(x) = 3x - 8\frac{1}{2}$ and $z(x) = \frac{3}{x-4} + 8$

$$y(x) = z(x)$$

$\therefore 3x - 8\frac{1}{2} = \frac{3}{x-4} + 8$

$\therefore 3x - 16\frac{1}{2} = \frac{3}{x-4}$

$\therefore \left(3x - 16\frac{1}{2}\right)(x-4) = 3$

$\therefore 3x^2 - 12x - 16\frac{1}{2}x + 66 - 3 = 0$

$\therefore 3x^2 - 28\frac{1}{2}x + 63 = 0$

$\therefore x^2 - 9\frac{1}{2}x + 21 = 0$

$\therefore 2x^2 - 19x + 42 = 0$

$\therefore (2x-7)(x-6) = 0$

$\therefore x = \frac{7}{2} = 3\frac{1}{2}$ or $x = 6$ Subs back into $y(x)$ for y:

$\therefore y = 3\left(3\frac{1}{2}\right) - 8\frac{1}{2}$ OR $y = 3(6) - 8\frac{1}{2}$

$\therefore y = 2$ $y = 9\frac{1}{2}$

$\therefore \left(3\frac{1}{2}; 2\right)$ and $\left(6; 9\frac{1}{2}\right)$

i) $b(x) = -4x$ and $c(x) = \frac{-1}{x+2} - 3$

$$b(x) = c(x)$$

$\therefore -4x = \frac{-1}{x+2} - 3$

$\therefore 3 - 4x = \frac{-1}{x+2}$

$\therefore (3-4x)(x+2) = -1$

$\therefore 3x + 6 - 4x^2 - 8x + 1 = 0$

$\therefore 4x^2 + 5x - 7 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-5 \pm \sqrt{(5)^2 - 4(4)(-7)}}{2(4)}$

$\therefore x = 0{,}84$ or $x = -2{,}09$ Subs back into $b(x)$ for y:

$\therefore y = -4(0{,}84)$ OR $y = -4(-2{,}09)$

$\therefore y = -3{,}35$ $y = 8{,}35$

$\therefore (0{,}84; -3{,}35)$ and $(-2{,}09; 8{,}35)$

j) $d(x) = \frac{1}{2}x + 7$ and $f(x) = 4x^2 - 6x + 4$

$d(x) = f(x)$

$\therefore \frac{1}{2}x + 7 = 4x^2 - 6x + 4$

$\therefore 0 = 4x^2 - 6\frac{1}{2}x - 3$

$\therefore 0 = 8x^2 - 13x - 6$

$\therefore 0 = (8x + 3)(x - 2)$

$\therefore \quad x = -\frac{3}{8} \quad or \quad x = 2$

Subs back into $d(x)$ for y:

$\therefore y = \frac{1}{2}\left(-\frac{3}{8}\right) + 7$ OR $y = \frac{1}{2}(2) + 7$

$\therefore y = 6\frac{13}{16}$ $\quad y = 8$

$\therefore \left(-\frac{3}{8}; 6\frac{13}{16}\right)$ $and\ (2; 8)$

7. a) $y = -x^2 + 2x - 3$

$\therefore y = -(x^2 - 2x + 3)$

$\therefore y = -(x^2 - 2x + 1 - 1 + 3)$

$\therefore y = -[(x - 1)^2 + 2]$

$\therefore y = -(x - 1)^2 - 2$

$\therefore$ Axis of symmetry: $x = 1$

b) $y = \frac{4}{x+6} + 8$

(-6; 8) and $m = \pm 1$

$\therefore y = -x + c$ and $y = x + c$

$\therefore 8 = -(-6) + c$ $\quad 8 = -6 + c$

$c = 2$ $\quad c = 14$

$\therefore$ Axes of Symmetry:

$\therefore y = -x + 2$ OR $y = x + 14$

c) $y = x^2 - 3x - 4$

$\therefore y = x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 4$

$\therefore y = \left(x - \frac{3}{2}\right)^2 - 6\frac{1}{4}$

$\therefore$ Axis of symmetry: $x = \frac{3}{2}$

d) $y = \frac{-1}{x+3} - 2$

(-3; -2) and $m = \pm 1$

$\therefore y = -x + c$ and $y = x + c$

$\therefore \ -2 = -(-3) + c$ $\quad -2 = -3 + c$

$\therefore c = -5$ $\quad c = 1$

$\therefore$Axes of symmetry:

$\therefore y = -x - 5$ OR $y = x + 1$

e) $y = x^2 - 8x + 15$

$\therefore \ y = x^2 - 8x + 16 - 16 + 15$

$\therefore y = (x - 4)^2 - 1$

$\therefore$ Axis of symmetry: $x = 4$

f) $y = 3^x$

No axis of symmetry.

g) $y = \frac{2}{x-1} + 4$ $\quad$ (1; 4) and $m = \pm 1$

$\therefore y = -x + c$ and $y = x + c$

$\therefore 4 = -(1)$+c $\quad 4 = 1 + c$

$\therefore 5 = c$ $\quad 3 = c$

$\therefore$Axes of Symmetry: $\therefore y = -x + 5$ $\quad y = x + 3$

h) $y = x + 2$

$\rightarrow$ no axis of symmetry

i) $y = -2x^2 + 7x - 9$

$\therefore y = -2\left(x^2 - \frac{7}{2}x + \frac{9}{2}\right)$

$\therefore y = -2\left(x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{9}{2}\right)$

$\therefore y = -2\left[\left(x - \frac{7}{4}\right)^2 + 1\frac{7}{16}\right]$

$\therefore y = -2\left(x - \frac{7}{4}\right)^2 - 2\frac{7}{8}$

$\therefore$ Axis of symmetry: $x = \frac{7}{4}$

j) $y = \frac{1}{x-1} + 3$

(1; 3) and $m = \pm 1$

$\therefore y = -x + c$ OR $y = x + c$

$\therefore 3 = -(1) + c$ $\quad 3 = 1 + c$

$\therefore c = 4$ $\quad \therefore c = 2$

$\therefore y = -x + 4$ OR $y = x + 2$

8. a) $y = -x^2 + 2x - 3$

$\therefore y = -(x^2 - 2x + 3)$

$\therefore y = -(x^2 - 2x + 1 - 1 + 3)$

$\therefore y = -[(x-1)^2 + 2]$

$\therefore y = -(x-1)^2 - 2$

$\therefore$ TP → (1; -2)

$\therefore$ maximum

b) $y = x^2 - 3x - 4$

$\therefore y = x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 4$

$\therefore y = \left(x - \frac{3}{2}\right)^2 - 6\frac{1}{4}$

$\therefore$ TP → $\left(\frac{3}{2}; -6\frac{1}{4}\right)$

$\therefore$ minimum

c) $y = x^2 - 8x + 15$

$\therefore y = x^2 - 8x + 16 - 16 + 15$

$\therefore y = (x-4)^2 - 1$

$\therefore$ TP → (4; -1)

$\therefore$ minimum

d) $y = -2x^2 + 7x - 9$

$\therefore y = -2\left(x^2 - \frac{7}{2}x + \frac{9}{2}\right)$

$\therefore y = -2\left(x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{9}{2}\right)$

$\therefore y = -2\left[\left(x - \frac{7}{4}\right)^2 + 1\frac{7}{16}\right]$

$\therefore y = -2\left(x - \frac{7}{4}\right)^2 - 2\frac{7}{8}$

$\therefore$ TP → $\left(\frac{7}{4}; -2\frac{7}{8}\right)$

$\therefore$ maximum

e) $y = x^2 + 8x + 9$

$\therefore y = x^2 + 8x + 16 - 16 + 9$

$\therefore y = (x+4)^2 - 7$

$\therefore$ TP → (-4; -7)

$\therefore$ minimum

f) $y = 2x^2 + 5x - 6$

$\therefore y = 2\left(x^2 + \frac{5}{2}x - 3\right)$

$\therefore y = 2\left(x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 - 3\right)$

$\therefore y = 2\left[\left(x + \frac{5}{4}\right)^2 - 4\frac{9}{16}\right]$

$\therefore y = 2\left(x + \frac{5}{4}\right)^2 - 9\frac{1}{8}$

$\therefore$TP → $\left(-\frac{5}{4}; -9\frac{1}{8}\right)$ $\quad \therefore$ minimum

g) $y = -3x^2 + 10x - 9$

$\therefore y = -3\left(x^2 - \frac{10}{3}x + 3\right)$

$\therefore y = -3\left(x^2 - \frac{10}{3}x + \left(\frac{10}{6}\right)^2 - \left(\frac{10}{6}\right)^2 + 3\right)$

$\therefore y = -3\left[\left(x - \frac{5}{3}\right)^2 + \frac{2}{9}\right]$

$\therefore y = -3\left(x - \frac{5}{3}\right)^2 - \frac{2}{3}$

$\therefore$ TP $\rightarrow \left(\frac{5}{3}; -\frac{2}{3}\right)$

$\therefore$ maximum

h) $y = 5x^2 - 9x + 4$

$\therefore y = 5\left(x^2 - \frac{9}{5}x + \frac{4}{5}\right)$

$\therefore y = 5\left(x^2 - \frac{9}{5}x + \left(\frac{9}{10}\right)^2 - \left(\frac{9}{10}\right)^2 + \frac{4}{5}\right)$

$\therefore y = 5\left[\left(x - \frac{9}{10}\right)^2 - \frac{1}{100}\right]$

$\therefore y = 5\left(x - \frac{9}{10}\right)^2 - \frac{1}{20}$

$\therefore$ TP $\rightarrow \left(\frac{9}{10}; -\frac{1}{20}\right)$

$\therefore$ minimum

i) $y = -\frac{1}{2}x^2 - 3x - 2$

$\therefore y = -\frac{1}{2}(x^2 + 6x + 4)$

$\therefore y = -\frac{1}{2}(x^2 + 6x + 9 - 9 + 4)$

$\therefore y = -\frac{1}{2}[(x + 3)^2 - 5]$

$\therefore y = -\frac{1}{2}(x + 3)^2 + \frac{5}{2}$

$\therefore$ TP $\rightarrow \left(-3; \frac{5}{2}\right)$

$\therefore$maximum

j) $y = \frac{1}{4}x^2 + \frac{1}{2}x + 1$

$\therefore y = \frac{1}{4}(x^2 + 2x + 4)$

$\therefore y = \frac{1}{4}(x^2 + 2x + 1 - 1 + 4)$

$\therefore y = \frac{1}{4}[(x + 1)^2 + 3]$

$\therefore y = \frac{1}{4}(x + 1)^2 + \frac{3}{4}$

$\therefore$ TP $\rightarrow \left(-1; \frac{3}{4}\right)$

$\therefore$ minimum

9. a) $y = a(x - p)^2 + q$ Subs in A (1; 2) (Turning Point)

$\therefore y = a(x - 1)^2 + 2$ Subs in B (-2; 1)

$\therefore 1 = a(-2 - 1)^2 + 2$

$\therefore -1 = 9a$

$\therefore a = -\frac{1}{9}$

$\therefore y = -\frac{1}{9}(x - 1)^2 + 2$

$\therefore y = -\frac{1}{9}(x^2 - 2x + 1) + 2$

$\therefore y = -\frac{1}{9}x^2 + \frac{2}{9}x - \frac{1}{9} + 2$

$\therefore y = -\frac{1}{9}x^2 + \frac{2}{9}x + 1\frac{8}{9}$

b) $m = -\frac{1}{4}$

$\therefore y = -\frac{1}{4}x + c$ Subs in B (-2; 1)

$\therefore 1 = -\frac{1}{4}(-2) + c$

$\therefore c = \frac{1}{2}$

$\therefore y = -\frac{1}{4}x + \frac{1}{2}$

c) $y = -\frac{1}{9}x^2 + \frac{2}{9}x + 1\frac{8}{9} \ldots 1$ and $y = -\frac{1}{4}x + \frac{1}{2} \ldots 2$

Subs 2 into 1:

$\therefore\ -\frac{1}{4}x + \frac{1}{2} = -\frac{1}{9}x^2 + \frac{2}{9}x + 1\frac{8}{9}$ (multiply out by LCD: 36)

$\therefore\ -9x + 18 = -4x^2 + 8x + 68$

$\therefore 4x^2 - 17x - 50 = 0$

$\therefore (x+2)(4x-25) = 0$

$\therefore x = -2 \quad or \quad x = \frac{25}{4} = 6\frac{1}{4}$

B $\therefore y = -\frac{1}{4}\left(6\frac{1}{4}\right) + \frac{1}{2}$

$\therefore y = -1\frac{1}{16}$ $\therefore$ D $\left(6\frac{1}{4};\ -1\frac{1}{16}\right)$

d) For $f(x) = -\frac{1}{9}x^2 + \frac{2}{9}x + 1\frac{8}{9}$

Let $y = 0$

$\therefore 0 = -\frac{1}{9}x^2 + \frac{2}{9}x + 1\frac{8}{9}$

$\therefore 0 = x^2 - 2x - 17$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = 5{,}24 \quad or \quad x = -3{,}24$

Let $x = 0$

$y = -\frac{1}{9}(0)^2 + \frac{2}{9}(0) + 1\frac{8}{9}$

$y = 1\frac{8}{9}$

$\therefore (5{,}24; 0), (-3{,}24; 0)\ and\ \left(0; 1\frac{8}{9}\right)$

For $g(x) = -\frac{1}{4}x + \frac{1}{2}$

Let $y = 0$

$\therefore 0 = -\frac{1}{4}x + \frac{1}{2}$

$\therefore \frac{1}{4}x = \frac{1}{2}$

$\therefore x = 2$

Let $x = 0$

$\therefore y = -\frac{1}{4}(0) + \frac{1}{2}$

$\therefore y = \frac{1}{2}$

$\therefore (2; 0)\ and\ \left(0; \frac{1}{2}\right)$

e) $-2 \le x \le 6\frac{1}{4}$

f) When $x = 0$:

Then $d_{fg} = 1\frac{8}{9} - \frac{1}{2}$

$d_{fg} = 1\frac{7}{18}$

g) $d = -\frac{1}{9}x^2 + \frac{2}{9}x + 1\frac{8}{9} - \left(-\frac{1}{4}x + \frac{1}{2}\right)$

$\therefore d = -\frac{1}{9}x^2 + \frac{2}{9}x + 1\frac{8}{9} + \frac{1}{4}x - \frac{1}{2}$

$\therefore d = -\frac{1}{9}x^2 + \frac{17}{36}x + 1\frac{7}{18}$

$\therefore d = -\frac{1}{9}\left(x^2 - 4\frac{1}{4}x - 12\frac{1}{2}\right)$

$\therefore d = -\frac{1}{9}\left(x^2 - 4\frac{1}{4}x + \left(2\frac{1}{8}\right)^2 - \left(2\frac{1}{8}\right)^2 - 12\frac{1}{2}\right)$

$\therefore d = -\frac{1}{9}\left[\left(x - 2\frac{1}{8}\right)^2 - 17\frac{1}{64}\right]$

$\therefore d = -\frac{1}{9}\left(x - 2\frac{1}{8}\right)^2 + 1\frac{57}{64}$

$\therefore x = 2\frac{1}{8}$

h) $x = 1$

i) $m(x) = -\frac{1}{9}(-x)^2 + \frac{2}{9}(-x) + 1\frac{8}{9}$

$\therefore m(x) = -\frac{1}{9}x^2 - \frac{2}{9}x + 1\frac{8}{9}$

j) $m(x) = g(x)$

$\therefore\ -\frac{1}{9}x^2 - \frac{2}{9}x + 1\frac{8}{9} = -\frac{1}{4}x + \frac{1}{2}$

$\therefore\ -\frac{1}{9}x^2 + \frac{1}{36}x + 1\frac{7}{18} = 0$

$\therefore 4x^2 - x - 50 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{1 \pm \sqrt{(-1)^2 - 4(4)(-50)}}{2(4)}$

$\therefore x = 3{,}66 \quad or \quad x = -3{,}41$ Subs back into $g(x)$ for y:

$\therefore y = -\frac{1}{4}(3{,}66) + \frac{1}{2}$ OR $y = -\frac{1}{4}(-3{,}41) + \frac{1}{2}$

$\therefore y = -0{,}42$ $y = 1{,}35$

k) $m(x) = -\frac{1}{9}x^2 - \frac{2}{9}x + 1\frac{8}{9}$

Let $y = 0$

$\therefore 0 = -\frac{1}{9}x^2 - \frac{2}{9}x + 1\frac{8}{9}$

$\therefore 0 = x^2 + 2x - 17$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-17)}}{2(1)}$

$\therefore x = 3{,}24 \quad or \quad x = -5{,}24$

Let $x = 0$

$y = -\frac{1}{9}(0)^2 - \frac{2}{9}(0) + 1\frac{8}{9}$

$y = 1\frac{8}{9}$

$\therefore (3{,}24; 0), (-5{,}24; 0)\ and\ \left(0; 1\frac{8}{9}\right)$

TP → $y = -\frac{1}{9}x^2 - \frac{2}{9}x + 1\frac{8}{9}$

$\therefore y = -\frac{1}{9}(x^2 + 2x - 17)$

$\therefore y = -\frac{1}{9}(x^2 + 2x + 1 - 1 - 17)$

$\therefore y = -\frac{1}{9}[(x+1)^2 - 18]$

$\therefore y = -\frac{1}{9}(x+1)^2 + 2$TP → (-1; 2) l)

	$m(x)$	$f(x)$
y-intercept	$\left(0; 1\frac{8}{9}\right)$	$\left(0; 1\frac{8}{9}\right)$
x-intercept	(3,24; 0) and (-5,24; 0)	(-3,24; 0) and (5,24; 0)
Turning Point	(-1; 2)	(1; 2)

∴ The y-intercepts are the same but the x-intercepts and turning point for $f(x)$ have been reflected about the y-axis to give the x-intercepts and turning point for $m(x)$.

m) $m(x) = f(x)$

$\therefore\ -\frac{1}{9}x^2 - \frac{2}{9}x + 1\frac{8}{9} = -\frac{1}{9}x^2 + \frac{2}{9}x + 1\frac{8}{9}$

$\therefore\ -\frac{2}{9}x - \frac{2}{9}x = 0$

$\therefore x = 0$

10. a) $h(x) = \frac{k}{x-p} + q$ $\quad x = 1;\quad y = 3$ C(-1; 2)

$\therefore h(x) = \frac{k}{x-1} + 3$ $\quad$ Subs in C

$\therefore 2 = \frac{k}{-1-1} + 3$

$\therefore\ -1 = \frac{k}{-2}$

$\therefore k = 2$ $\quad \therefore h(x) = \frac{2}{x-1} + 3$

b) Let $x = 0$ $\quad \therefore y = \frac{2}{0-1} + 3$

$\therefore y = -2 + 3$

$\therefore y = 1$ $\quad$ ∴ A (0; 1)

c) $p(x) = mx + 1$ Subs in $\left(\frac{1}{2}; 3\right)$

$\therefore 3 = m\left(\frac{1}{2}\right) + 1$

$\therefore 2 = \frac{1}{2}m$

$\therefore m = 4$ $\quad \therefore p(x) = 4x + 1$

d) $h(x) = p(x)$

$\therefore \frac{2}{x-1} + 3 = 4x + 1$

$\therefore \frac{2}{x-1} = 4x - 2$

$\therefore 2 = (4x - 2)(x - 1)$

$\therefore 2 = 4x^2 - 4x - 2x + 2$

$\therefore 0 = 4x^2 - 6x$

$\therefore 0 = x(4x - 6)$

$\therefore x = 0 \quad or \quad x = \frac{3}{2}\ or\ 1\frac{1}{2}$

$\therefore y = 4\left(1\frac{1}{2}\right) + 1$

$\therefore y = 7$ $\quad \therefore B\ \left(1\frac{1}{2}; 7\right)$

e) $y = \pm x + c$ $\quad$ (1; 3)

$\therefore 3 = -(1) + c$ $\quad$ and $\quad 3 = 1 + c$

$\therefore c = 4$ $\quad c = 2$

$\therefore y = -x + 4$ $\quad y = x + 2$ $\quad$ A

f) $x = 1$

g) $h(x)$: when $x = 0$ then $y = 1$ (A)

Let $y = 0$

$\therefore 0 = \frac{2}{x-1} + 3$

$\therefore -3 = \frac{2}{x-1}$

$\therefore -3(x-1) = 2$

$\therefore -3x + 3 = 2$

$\therefore -3x = -1$

$\therefore x = \frac{1}{3}$

$p(x)$: when $x = 0$ then $y = 1$

Let $y = 0$ $\therefore 0 = 4x + 1$

$\therefore -1 = 4x$

$\therefore x = -\frac{1}{4}$ $\left(-\frac{1}{4}; 0\right)$

h) $0 \leq x < 1$

i) $h(x) \rightarrow$ C (-1; 2)

$p(-1) = 4(-1) + 1$

$p(-1) = -3$

Height $= 2 - (-3)$

$\therefore$ Height $= 5$ units