Measurement

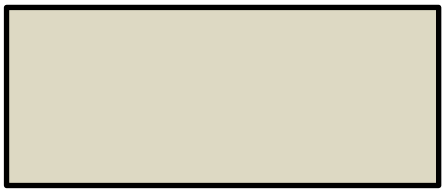
Grade 10 Mathematics

Grade 9 Revision

Area and Perimeter:

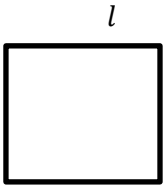
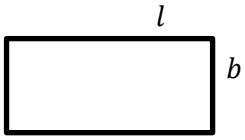
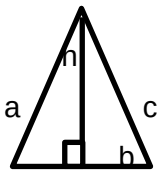
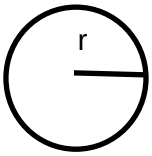
Perimeter is the distance around the shape – or how much ink it takes to draw the shape.

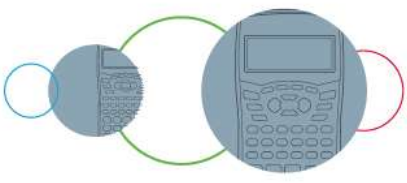
Area is the space the shape takes up on the page.

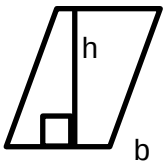
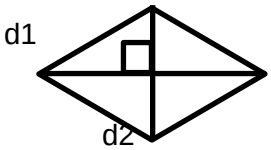
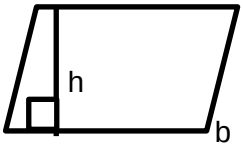
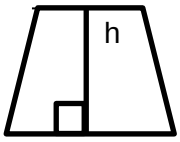


The black line around the rectangle is the perimeter, while the grey inside the rectangle is the area.

You need to know all the formulas for the area and perimeter of different shapes – so learn them off by heart. They are summarised below for you:

Shape	Picture	Perimeter	Area
Square		$P = 4l$	$A = l^2$
Rectangle		$P = 2(l + b)$	$A = l \times b$
Triangle		$P = a + b + c$	$A = \frac{1}{2}(b \times h)$
Circle		$P = 2\pi r$ Also known as Circumference	$A = \pi r^2$



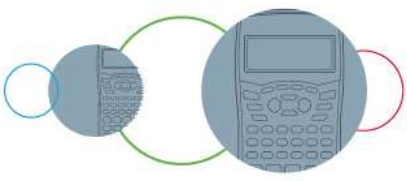
Rhombus		P = sum of all four sides P = 4b	$A = b \times h$
Kite		P = sum of all four sides	$A = \frac{1}{2}(diagonal_1 \times diagonal_2)$
Parallelogram		P = sum of all four sides	$A = b \times h$
Trapezium	Parallel side 1  Parallel side 2	P = sum of all four sides	$A = \frac{1}{2}(sum\ of \parallel sides) \times h$

Units:

Please remember that all the units on your diagram need to be the same. Watch out for this as teachers will often mix units in questions.

Make sure that you can convert between the different units of measurement:

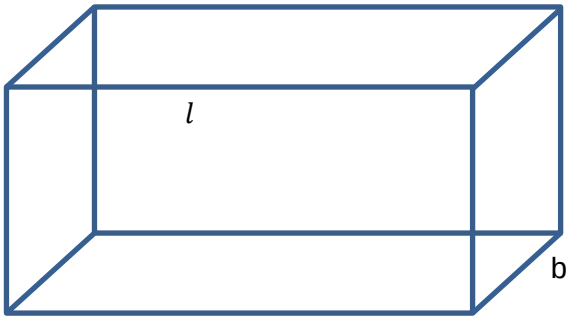
Name	*kilo*	*hecto*	*deca*	*meter*	*deci*	*centi*	*milli*
Abbreviation	k	h	da	m	d	c	m
Value	10^3 or 1000	10^2 or 100	10^1 or 10	10^0 or 1	10^{-1} or 0,1	10^{-2} or 0,01	10^{-3} or 0,001
Rhyme	King	Henry	died	(by)	drinking	chocolate	milk



Surface Area, Volume and Capacity:

Think about a rectangular tissue box or a lunch box.

How many faces (or sides) does the tissue box have? If you counted 6 you are correct. Now look at the box again and see how many sides are identical to each other. You should see that there are 3 sets of 2 sides each that match each other perfectly. They are opposite each other in the box. Each face or side has a particular area. For example the top side has an area of $l \times b$, while the side area is $b \times h$. The area in the front of the box is $l \times h$. Remember that we have 2 of each type of face so the formula for the Surface area of the rectangular prism is:



$$\boldsymbol{SA = 2l \times b + 2b \times h + 2l \times h}$$

Remember: *Surface area* is the area of the surface (or outside faces) of the prism.

Volume is the amount of space a 3D shape takes up or occupies. Think about a book or a stack of paper. A single sheet of paper would represent a 2D or flat rectangle and the area of the sheet of paper would be $length \times breadth \; or \; l \times b$. If we start to stack another sheet of paper on top and continue to add more paper to the stack of paper, the amount of space taken up by the stack of paper will increase. How do we measure the volume? Well let's pretend that each sheet of paper is exactly 1mm thick. So, to measure the volume of 2 sheets of paper we would times the base area ($l \times b$) by 2, if the stack is 3 sheets of paper we would multiply the base area by 3 and so on. Thus the more sheets we add the greater the volume of paper. If we have h sheets of paper then we would say $volume = (l \times b) \times h$.

In maths we say that h is the height of the prism. Therefore the volume of a rectangular prism is

$$\boldsymbol{Volume = l \times b \times h}$$

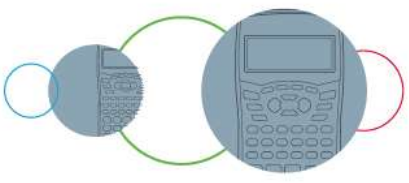
This method can be used to work out the volume of other 3D shapes such as cylinders and triangular prisms. In other words, multiply the base area of the shape (for example, the triangle or circle) by the height to get the volume.

Capacity is the amount of space inside a prism – in other words, how much water can the rectangular box hold? Capacity is measured in millilitres, litres and kilolitres and is related to volume by:

$$\boldsymbol{1ml = 1cm^3}$$

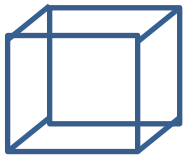
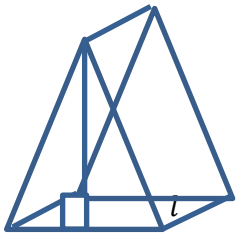
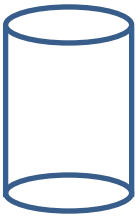
This means that 1 cubic centimetre (a cube measuring 1 cm by 1 cm by 1cm) has the capacity of 1ml. And

$$\boldsymbol{1kl = 1\,m^3}$$



A 1m by 1m by 1m cube has the capacity of 1 kilolitre or 1 000 litres.

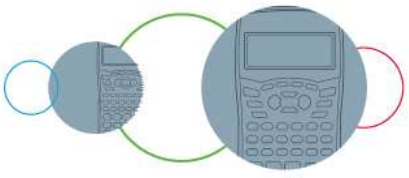
The following table gives a summary of the surface area and volume formulas for different prisms:

Name	Picture	Surface Area	Volume
Cube	Length = l all sides 	6 identical sides $\therefore SA$ $= 6(area\ of\ 1\ face)$ $\therefore SA = 6l^2$	$V = l^3$
Rectangular prism	Length = l, breadth = b height = h 	$SA = 2lb + 2lh + 2bh$	$V = lbh$
Triangular Prism		SA = area of rectangle 1 + area of rectangle 2 + area of rectangle 3 + 2(area of triangle)	V = area of triangle x length $V = \frac{1}{2}(b \times h) \times l$
Cylinder		SA = 2(area of circle) + area of rectangle.* $SA = 2\pi r^2 + 2\pi rh$	V = area of circle x height $V = \pi r^2 h$

* To work out the surface area of the cylinder, imagine a toilet roll (even better would be to have a toilet roll with you). If you look at the top and bottom of the toilet roll you will see the two circles mentioned. Now, make a mark where the end of the toilet paper begins on the next layer down and unwind the toilet paper until it reaches that mark. What shape does the toilet paper make?

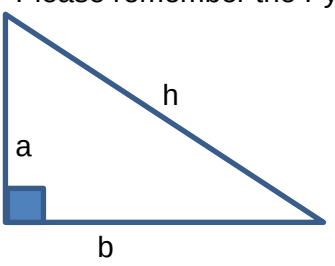
A rectangle!

The height of the rectangle is the height of the cylinder (or toilet roll) and the length of the toilet paper is the circumference of the toilet paper or $2\pi r$. Do you see it? Wind the toilet paper back around the toilet roll to see where the length comes from. This is how you find the surface area of a cylinder.



One more thing…

Please remember the Pythagoras rule: hypotenuse^2 = sum of the other two sides squared.



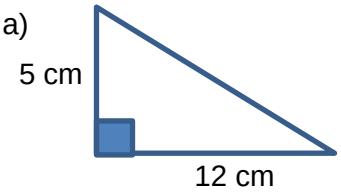
$$h^2 = a^2 + b^2$$

Remember that Pythagoras rule can only be used when there is a right angle.

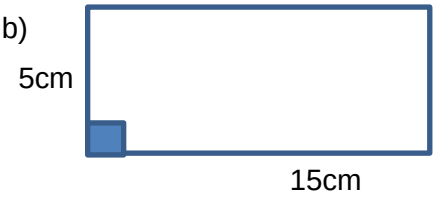
Exercise 1:

1. Find the area and perimeter of the following shapes

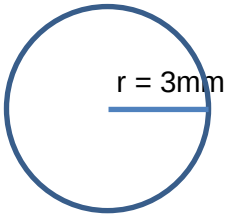
a)



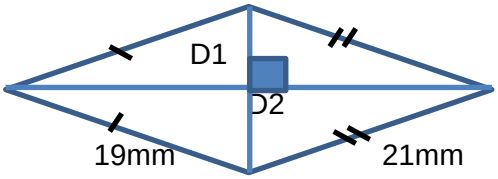
b)



c)



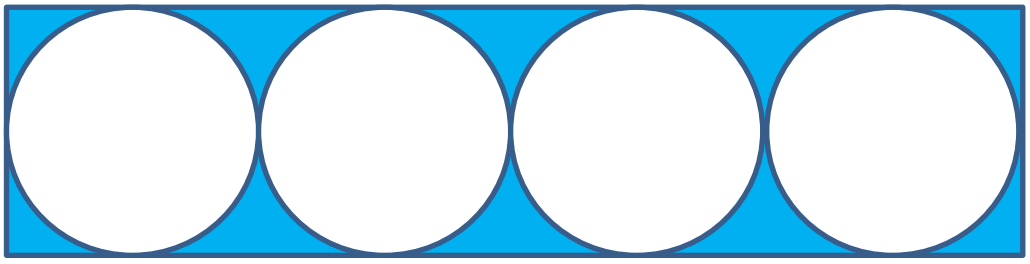
d)



diagonal 1 = 34mm; diagonal 2 = 12mm

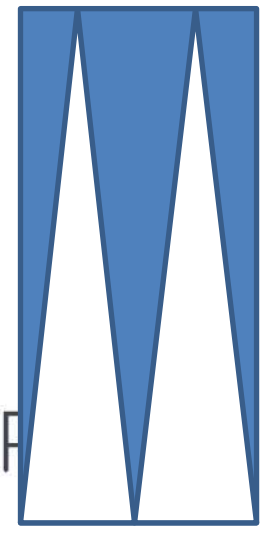
2. Find the shaded area:

a)



Diameter of circles = 3m.

b)



Rectangle:

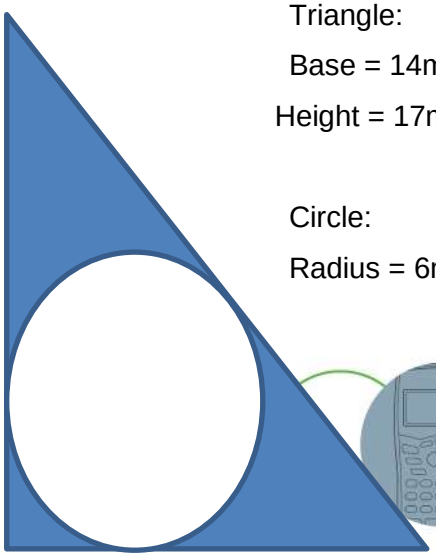
Base = 12cm

Height = 340mm

Triangles:

Base = 6cm

c)



Triangle:

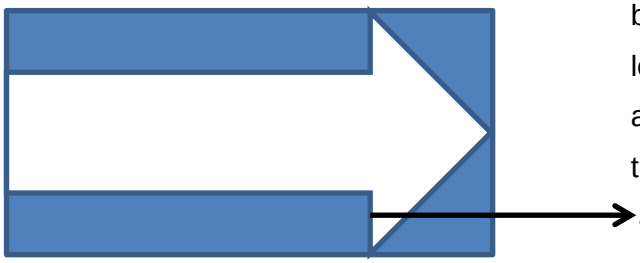
Base = 14m

Height = 17m

Circle:

Radius = 6m

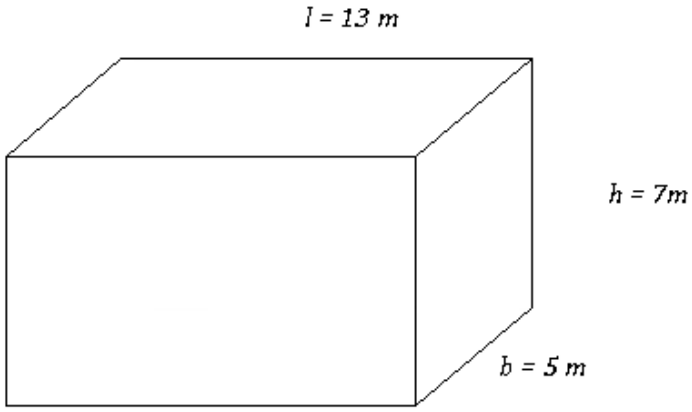
d) length of rectangle = 28 km; height of rectangle = 18km. The tip of the arrow bisects the height into two equal lengths. The base of the triangle and the height of the triangles are the same.



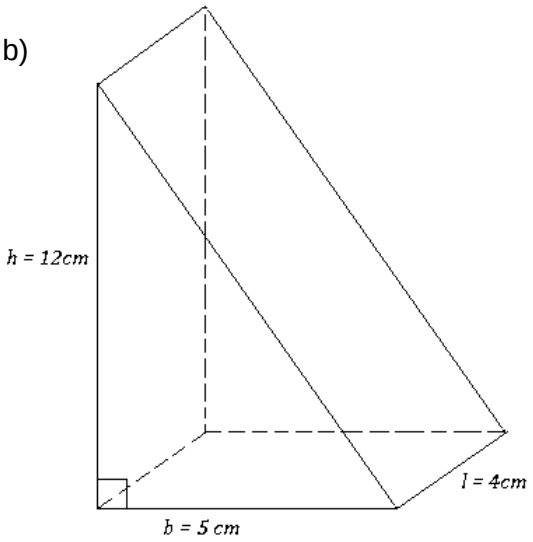
e) Rectangle:
Length = 75m
Height = 21

3. Find the volume and surface area of the following prisms:

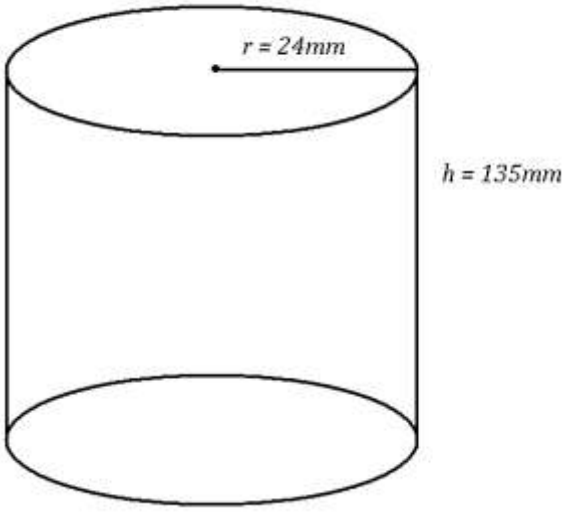
a)



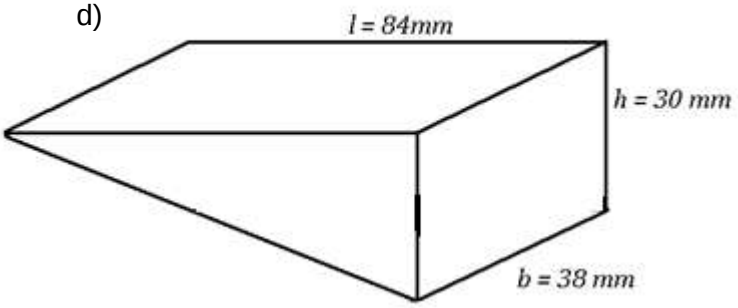
b)



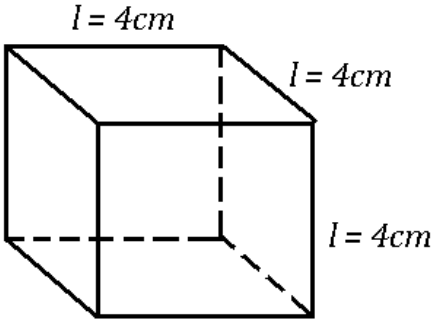
c)



d)



e)



4. Find the missing value of the following prisms:

a) A rectangular prism of volume 168 750m^3 with a square base and a height of 30m.

b) A cylinder with a surface area of 4266π mm^2 and a height of 52mm.

c) A cube with a volume of 166 375cm^3.

d) A rectangular prism with height 72mm, breadth 10 mm and surface area 7508mm^2

e) A cylinder with a volume of 19 375π cm^3, and a radius of 25cm.

f) A right angled triangular prism with volume 26730mm^3, base 33mm and length 60mm.

g) A cube with a surface area 726m^2.

h) A rectangular prism with length 16m, height 31m and volume 16 864m^3

i) A cylinder with a radius 5,2cm and surface area 76,1π cm^2

j) A right angles triangular prism with volume 2,795mm^3, base 4.3mm and length 1mm.

Effect of changing dimensions:

Let's think about a rectangular prism where the length = x, the breadth = y and the height = z

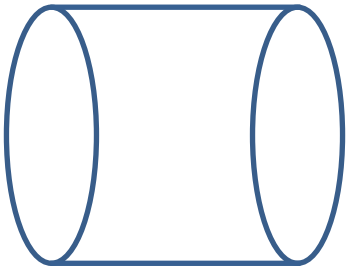
Now the volume will be given by $V = xyz$ and the surface area will be given by $SA = 2xy + 2xz + 2yz$.

What happens if we double the length? In other words – length becomes $2x$. Now our Volume will become $V = 2xyz$ or twice as much as our original volume. If we multiplied x (our length) by some factor, k, so that our length is now kx, then the volume will become $V = kxyz$ or in other words, the volume is multiplied by the factor k.

If we also multiplied our breadth, y by k then we would have $V = kx \times ky \times z = k^2xyz$ – now you can see that the factor k has been squared. What do you think will happen if we multiplied each dimension by k?

That's right your volume will now look like this: $V = kx \times ky \times kz = k^3xyz$ or k^3 times more than your original volume.

This works well for rectangular prisms and cubes, but what happens with cylinders?



The formula for the volume of a cylinder is $V = \pi r^2 h$. If we multiply the height by the constant k, our volume will now be $V = \pi r^2(kh) = k\pi r^2 h$ or k times our original volume. Now look at what happens to our volume when we multiply the radius by k. Because the radius in the volume formula is squared it means that the factor k is also squared. So now our volume will become:

$V = \pi(kr)^2 h = k^2 \pi r h$ or k^2 times our original volume.

This process of substitution works for any form of prism and for both surface area and volume. Remember to substitute the original values and the increase or decrease factor separately – this will make it easier to see the original volume (or surface area) and how much it has been increased or decreased by.

Example: If a cylinder has a radius of $x$cm and a height of $y$cm what will happen to the volume if:

a) the radius is doubled?

b) the height is tripled?

c) both the radius and the height are halved?

Answers:

a) original volume = $\pi r^2 h = \pi(x^2)y$

Radius doubled means $x \times 2$

∴ new volume = $\pi r^2 h = \pi(2x)^2 y = 4\pi x^2 y$

∴ the volume is quadrupled in size.

b) height tripled means $y \times 3$

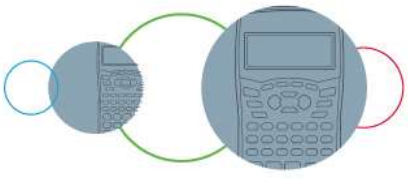
∴ new volume = $\pi r^2 h = \pi(x)^2(3y) = 3\pi x^2 y$

∴ the volume has tripled in size.

c) radius halved = $\frac{x}{2}$ and height halved = $\frac{y}{2}$

∴ new volume = $\pi r^2 h = \pi\left(\frac{x}{2}\right)^2\left(\frac{y}{2}\right) = \frac{1}{8}\pi x^2 y$

∴ the volume is one eight of its original size



Exercise 2:

1. Given a cube with length 4m, what will happen to the volume if
   a) the length is doubled? b) the length is tripled?
   c) the length is halved?

2. Given a cube with length 3cm, what will happen to the surface area if all three sides are
   a) doubled? b) tripled?
   c) halved?

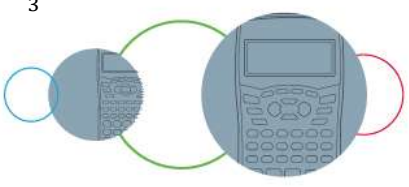
3. Given a rectangular prism with length 5mm, height 6mm and breadth 7mm, what will happen to
   a) the volume if:
      i) the length is doubled?
      ii) both the length and breadth are doubled?
      iii) the height is tripled?
      iv) the height is doubled and the length is halved?

   b) the surface area if:
      i) the length is multiplied by a factor of $\frac{3}{2}$?
      ii) the breadth and height are increased by a factor of 4?
      iii) the length and height are decreased by a factor of 2?

4. Given a cylinder with height, h and radius r, determine what will happen to the volume if:
   a) the height is doubled and the radius is halved?
   b) the height is halved and the radius is doubled?
   c) the radius is tripled?
   d) the height is tripled and the radius is decreased by a factor of 3?

5. a) Given the original volume of a cylinder is 500π m^3, determine the new volume if:
      i) the radius is doubled. ii) the height is doubled.
      iii) the radius is halved. iv) the height is halved.
      v) the radius is tripled and the height is halved.

   b) Given the original volume of a rectangular prism is 460cm^3, determine the new volume if:
      i) the length, breadth and height are doubled.
      ii) the length and breadth are doubled and the height is halved.
      iii) the length and breadth are halved and the height is doubled.
      iv) the length, breadth and height are halved.
      v) the length, breadth and height are increased by a factor of $\frac{5}{3}$.



c) Given the original volume of a cube is 125mm^3, determine the new volume if:

i) the lengths are doubled. ii) the lengths are tripled.

iii) the lengths are halved.

iv) the lengths are decreased by a factor of $\frac{2}{5}$.

d) Given the original surface area of a rectangular prism is 400mm^2, determine the new surface area if:

i) the length, breadth and height are halved

ii) All three dimensions are increased by a factor of $\frac{3}{2}$.

e) Given the original surface area of a cylinder is 51.6 cm^2, determine the new surface area if:

i) the radius and the height are tripled.

ii) the height and the radius are doubled.

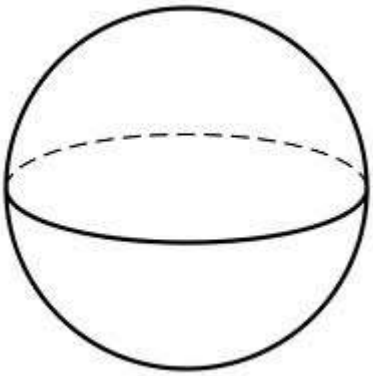
iii) the height and the radius are halved.

iv) the height and the radius are decreased by a factor of $\frac{3}{2}$.

Volume and Surface Area of Spheres, Right Pyramids and Cones

Sphere

A sphere is a 3D circle or a ball as you can see from the picture on the right.



The volume of a sphere is given by the formula:

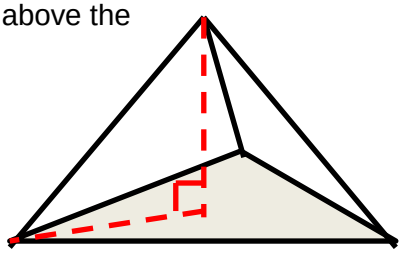
$$\boldsymbol{V = \frac{4}{3}\pi r^3}$$

And the surface area of a sphere is given by the formula:

$$\boldsymbol{SA = 4\pi r^2}$$

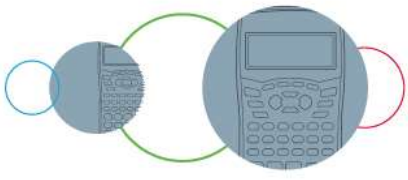
Right Pyramid

A pyramid has a base and an apex. The base is the shape at the bottom of the pyramid – for example, an equilateral triangle or a square. The apex is the highest point above the pyramid. The height is the distance from the base to the apex, and is at a right angle to the base.



The volume of a pyramid can be found using the following general formula:

$$\boldsymbol{V = \frac{1}{3}(area\ of\ base) \times height}$$



And the Surface area of a right pyramid can be found using the general formula:

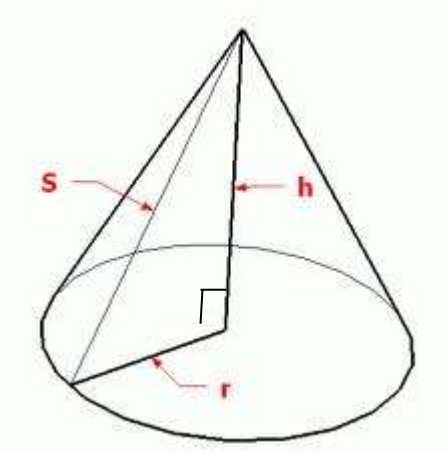
$$\boldsymbol{SA = (area\ of\ base) + \frac{1}{2}(perimeter\ of\ the\ base) \times the\ slant\ height}$$

OR

$$\boldsymbol{SA = area\ of\ base + (number\ of\ triangle\ faces) \times (area\ of\ triangle\ face)}$$

The second formula will only work if all the triangle faces are identical – in other words – the base is a regular polygon (a shape where all the sides are equal).

Right Cone



A right cone, is a right pyramid where a circle is the base. This means that the apex is directly above the centre of the circle. In this picture s is the slant height – to find it use your knowledge of Pythagoras to find the length of the hypotenuse.

The volume of a cone can be found using the formula:

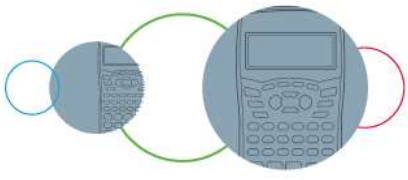
$$\boldsymbol{V = \frac{1}{3}\pi r^2 h}$$

And the surface area of the cone can be found using the formula:

$$\boldsymbol{SA = \pi r^2 + \pi r s}$$

Exercise 3

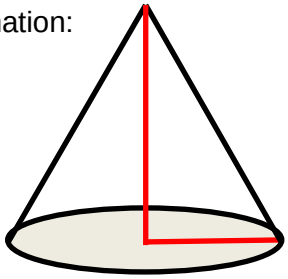
1. Find the volume and surface area of the following shapes:
   a) A sphere with a radius of 2cm.
   b) A right square pyramid, with the length of one side of the square being 4cm and the height from the base to the apex is 5cm.
   c) A right cone with a radius of 55mm and a height of 70mm.
   d) A sphere with a radius of 16mm.
   e) A right equilateral triangle pyramid with the length of one side of the equilateral equal to 15cm and the height equal to 8cm. The distance from the centre of the pyramid (where the height is measured) to the midpoint of one side of the triangle is 6cm.
   f) A right cone with a radius of 3.4cm and a height of 11cm.
   g) A right circular pyramid with a radius of 14mm and a height of 17mm
   h) A sphere with a radius of 59mm.
   i) A right square pyramid with the length of one side of the square as 30mm and a height of 45mm.
   j) A right square pyramid with the length of one side of the square being 21cm and the height of the pyramid being 25cm.



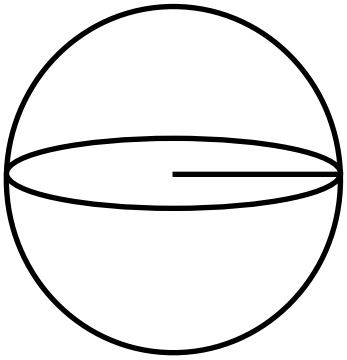
2. Given the volume of each shape find the missing information:

a) V = $8008\frac{1}{3}\pi$ cm^3, with radius = 31cm.

Find the height and the surface area.



b)



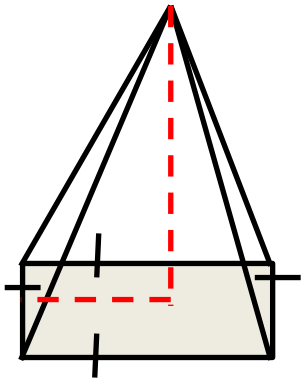
V = 3705.97mm^3

Find the radius and the surface area of the sphere.

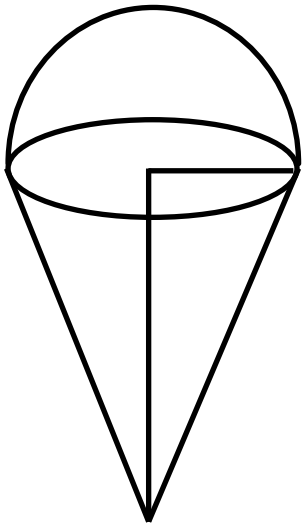
c) V = 14.583333 cm^3, with height = 7cm.

Find the length of one side of the square,

the slant height and the surface area.



d)

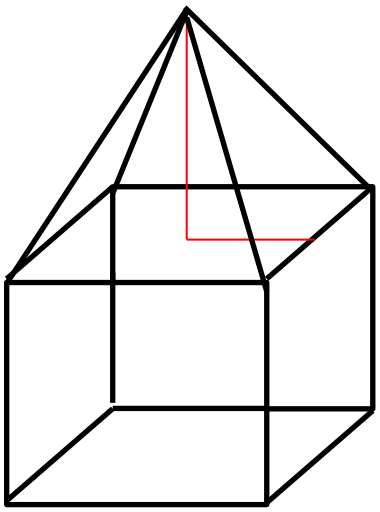


The cone and half sphere on top have a volume of

V = $74\ 666\frac{2}{3}\pi$ mm^3, and a radius of 40mm.

Find the height of the cone, and find the surface area of the cone and sphere.

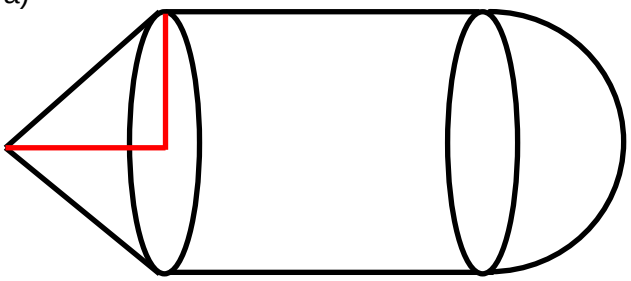
e) The volume of the cube and right pyramid are given as V = 518.689333cm^3.



The height of the pyramid is the same as the length of the side of the cube.

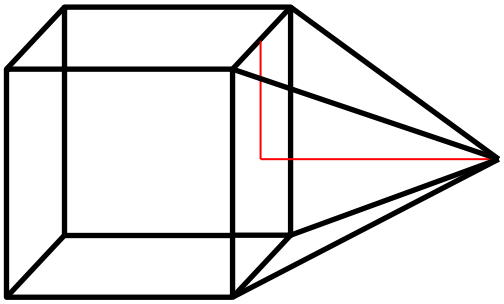
Find the height and length of the cube and right pyramid, and find the surface area of the shape as well.

3. a)



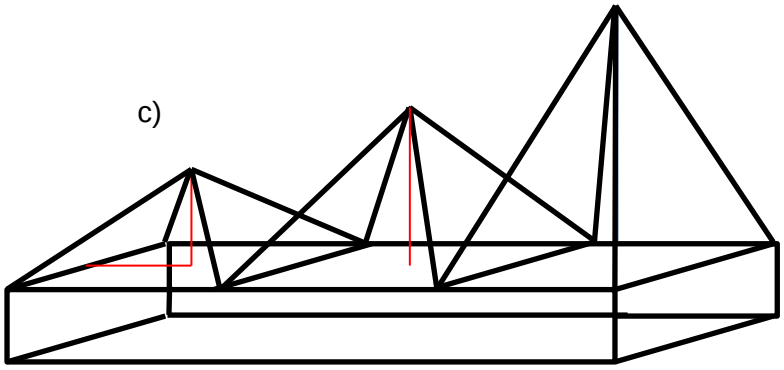
The cylinder has a radius of 8cm, and a length of the cylinder is twice as long as the radius. The cone has a height that is $\frac{3}{2}$ of the cylinder's radius. Determine the volume and surface area of the shape.

b)



Given in the picture is a cube with length x, and a right pyramid, with height x. Given that the surface area of the shape is equal to 1 575cm^{2,} determine the value for x and the volume of the shape.

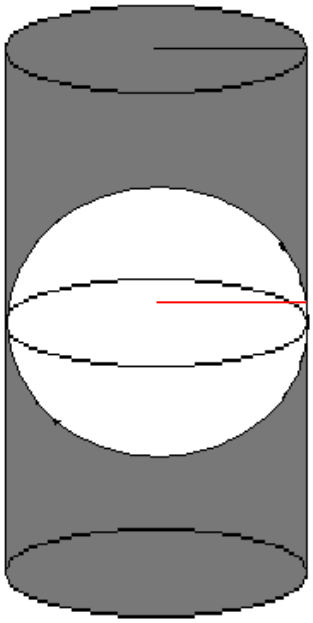
c)



The base of the triangles is a Rectangular prism, with length 27cm, breadth 9cm and height 3cm. The height of the first pyramid is 3cm, the height of the second pyramid is 6cm and the height of the third pyramid is 9cm, determine the surface area and volume of the shape.

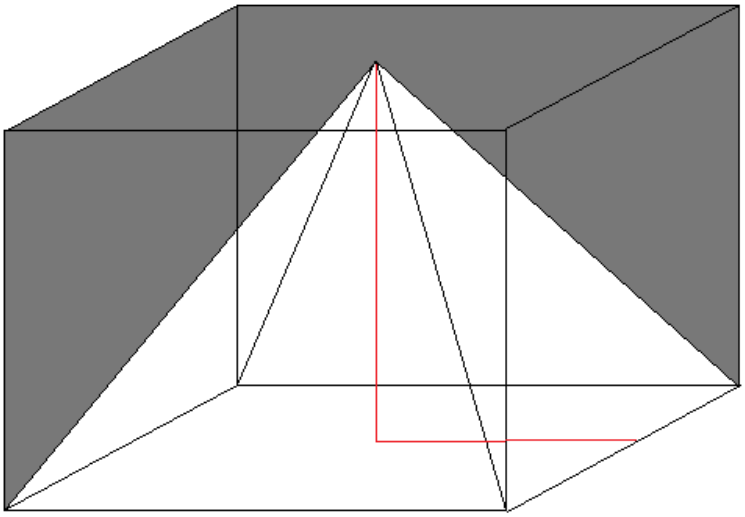
4. Find the volume occupied by of the shaded regions.

a)



Radius = 4cm, height = 12cm

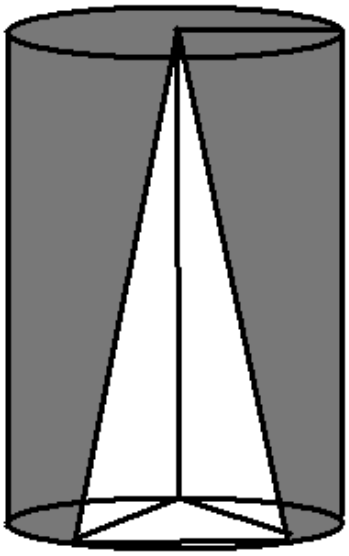
b)



Length = 15cm, breadth = 15cm

Height = 20cm

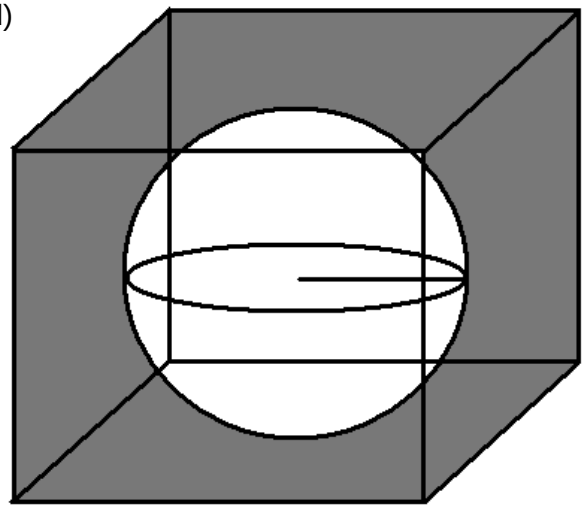
c)



Radius = 5mm; height = 16mm

Length of triangle side = 8mm

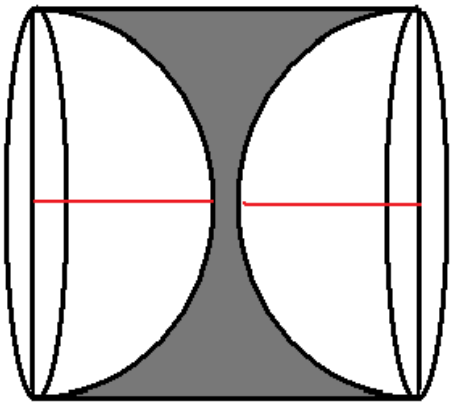
d)



Length = 44mm, breadth = 79mm

height = 40mm, Radius = 20mm

e)



Radius = 90mm

Height = 188mm.