SHARP

Worksheet 8 Memorandum: Solving Equations

Grade 10 Technical Mathematics

1. a) $3x + 7 = 12$

$3x = 5$

$x = \frac{5}{3}$

b) $3(x + 7) - 4 = 2(x - 2)$

$3x + 21 - 4 = 2x - 4$

$3x - 2x = -4 + 4 - 21$

$x = -21$

c) $\frac{4}{x} + 2 = 8$

$\frac{4}{x} = 6$

$4 = 6x$

$x = \frac{4}{6}\ or \frac{2}{3}$

d) $m(m + 3) - 7 = m^2 + 2m - 1$

$m^2 + 3m - 7 = m^2 + 2m - 1$

$m = -1 + 7$

$m = 6$

e) $\frac{p+3}{5} - 8 = 1$

$\frac{p+3}{5} = 9$

$p + 3 = 45$

$p = 42$

f) $\frac{1}{2}(q + 3) - 4 = \frac{1}{4}(q - 3)$

$\frac{1}{2}q + \frac{3}{2} - 4 = \frac{1}{4}q - \frac{3}{4}$

$\frac{1}{2}q - \frac{1}{4}q = -\frac{3}{4} + 4 - \frac{3}{2}$

$\frac{1}{4}q = 1\frac{3}{4}$

$q = 7$

g) $\frac{y}{3} + \frac{y+1}{2} = 7$

$\frac{2y}{6} + \frac{3y+3}{6} = 7$

$5y + 3 = 42$

$5y = 39$

$y = \frac{39}{5}\ or\ 7\frac{4}{5}$

h) $\frac{1}{x} + 8 = 3$

$\frac{1}{x} = -5$

$1 = -5x$

$x = -\frac{1}{5}$

i) $t(t+4)-5t=t(6+t)-8$

$t^2+4t-5t=6t+t^2-8$

$-t-6t=\ -8$

$-7t=\ -8$

$t=\frac{8}{7}\ or\ 1\frac{1}{7}$

j) $(m+3)(m-6)=(m-4)(m-7)$

$m^2-3m-18=m^2-11m+28$

$-3m+11m=28+18$

$8m=46$

$m=\frac{46}{8}\ \ or\ \ 5\frac{3}{4}$

2. a) $\frac{y+4}{3}-\frac{2y-3}{5}=\frac{y}{15}$

$\frac{5y+20}{15}-\frac{6y-9}{15}=\frac{y}{15}$

$5y+20-6y+9=y$

$5y-6y-y=\ -20-9$

$-2y=\ -29$

$y=\frac{29}{2}\ \ or\ 14\frac{1}{2}$

b) $\frac{3y+1}{4}-\frac{\frac{3}{4}y^2-1}{y}=7$

$\frac{3y^2+y}{4y}-\frac{3y^2-4}{4y}=\frac{28y}{4y}$

$3y^2+y-3y^2+4=28y$

$y-28y=\ -4$

$-27y=\ -4$

$y=\frac{4}{27}$

c) $\frac{(y+3)(y-9)}{y+1}=\frac{3y-7}{3}$

$\frac{3(y^2-6y-27)}{3(y+1)}=\frac{(3y-7)(y+1)}{3(y+1)}$

$3y^2-18y-81=3y^2-4y-7$

$-18y+4y=\ -7+81$

$-14y=74$

$y=-5\frac{2}{7}$

d) $\frac{m-9}{m+3}-\frac{2m+8}{m-5}+1=0$

$\frac{(m-9)(m-5)-(2m+8)(m+3)+(m+3)(m-5)}{(m+3)(m-5)}=0$

$m^2-14m+45-(2m^2+14m+24)+m^2-2m-15=0$

$2m^2-16m+30-2m^2-14m-24=0$

$-30m+6=0$

$-30m=\ -6$

$m=\frac{1}{5}$

e) $\frac{2a-7}{b} - \frac{3a-4}{4} = 7b$ solve for a in terms of b.

$\frac{8a-28}{4b} - \frac{3ab-4b}{4b} = \frac{28b^2}{4b}$

$8a - 28 - 3ab + 4b = 28b^2$

$8a - 3ab = 28b^2 + 28 - 4b$

$a(8 - 3b) = 28b^2 - 4b + 28$

$\therefore a = \frac{28b^2-4b+28}{8-3b}$

3. a) $x^2 + 14x - 240 = 0$

$(x - 10)(x + 24) = 0$

$\therefore x - 10 = 0 \ \ or \ x + 24 = 0$

$\therefore x = 10 \quad or \ x = -24$

b) $x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$

$\therefore x - 5 = 0 \ \ or \ x - 2 = 0$

$\therefore x = 5 \quad or \ x = 2$

c) $x(x + 7) = -4(2x + 11)$

$x^2 + 7x = -8x - 44$

$x^2 + 15x + 44 = 0$

$(x + 4)(x + 11) = 0$

$\therefore x = -4 \ or \ x = -11$

d) $m^2 - 19 = 5(m + 1)$

$m^2 - 19 = 5m + 5$

$m^2 - 5m - 24 = 0$

$(m - 8)(m + 3) = 0$

$m = 8 \ \ or \ m = -3$

e) $(p + 6)(p + 1) = 24$

$p^2 + 7p + 6 = 24$

$p^2 + 7p - 18 = 0$

$(p + 9)(p - 2) = 0$

$p = -9 \ or \ p = 2$

f) $m(m + 4) = 4(m + 4)$

$m^2 + 4m = 4m + 16$

$m^2 - 16 = 0$

$(m - 4)(m + 4) = 0$

$m = 4 \ \ or \ m = -4$

g) $t(t-15) = 12\left(t-10\frac{1}{2}\right)$

$t^2 - 15t = 12t - 126$

$t^2 - 27t + 126 = 0$

$(t-6)(t-21) = 0$

$t = 6 \;\; or \;\; t = 21$

h) $(w-7)(w+7) = 3(5-4w)$

$w^2 - 49 = 15 - 12w$

$w^2 + 12w - 64 = 0$

$(w+16)(w-4) = 0$

$w = -16 \;\; or \;\; w = 4$

i) $(y-3)(y-15) = -20$

$y^2 - 18y + 45 = -20$

$y^2 - 18y + 65 = 0$

$(y-13)(y-5) = 0$

$y = 13 \;\; or \;\; y = 5$

j) $(a+3)(a+1) = 2(69-a)$

$a^2 + 4a + 3 = 138 - 2a$

$a^2 + 6a - 135 = 0$

$(a-9)(a+15) = 0$

$a = 9 \;\; or \;\; a = -15$

4. a) $y = 3x - 5$ 1 and $x = 5y - 3$ 2

Subs 1 into 2:

$x = 5(3x-5) - 3$

$x = 15x - 25 - 3$

$-14x = -28$

$x = 2$

Subs back into 1

$y = 3(2) - 5$

$y = 6 - 5$

$y = 1$

b) $0 = 2x + 7y$ 1 and $8 = 2x - y$

$y = 2x - 8$ 2

Subs 2 into 1

$0 = 2x + 7(2x-8)$

$0 = 2x + 14x - 56$

$-16x = -56$

$x = 3\frac{1}{2}$

Subs back into 2

$y = 2\left(3\frac{1}{2}\right) - 8$

$y = 7 - 8$

$y = -1$

c) $\frac{m+n}{3} = 4$ and $3m - 2n = 8$ 2

$m + n = 12$

$m = 12 - n$ 1

Subs back into 1

$m = 12 - 5\frac{3}{5}$

$m = 6\frac{2}{5}$

Subs 1 into 2:

$3(12 - n) - 2n = 8$

$36 - 3n - 2n = 8$

$-5n = -28$

$n = 5\frac{3}{5}$

d) $-\frac{1}{3}m - 2n = 5$ and $-\frac{1}{2}n + 2m + 3 = 0$ 2

$-\frac{1}{3}m = 5 + 2n$

$m = -15 - 6n$ 1

Subs back into 1

$m = -15 - 6\left(-2\frac{4}{25}\right)$

$m = -15 + 12\frac{24}{25}$

$m = -2\frac{1}{25}$

Subs 1 into 2

$-\frac{1}{2}n + 2(-15 - 6n) + 3 = 0$

$-\frac{1}{2}n - 30 - 12n + 3 = 0$

$-12\frac{1}{2}n = 27$

$n = -2\frac{4}{25}$

e) $2a + 8b = \frac{1}{3}$ and $\frac{1}{4}(a + 12b) = -3$ 2

$2a = \frac{1}{3} - 8b$

$a = \frac{1}{6} - 4b$ 1

Subs back into 1

$a = \frac{1}{6} - 4\left(-1\frac{25}{48}\right) = 6\frac{1}{4}$

Subs 1 into 2

$\frac{1}{4}\left(\frac{1}{6} - 4b + 12b\right) = -3$

$\frac{1}{4}\left(\frac{1}{6} + 8b\right) = -3$

$\frac{1}{24} + 2b = -3$

$2b = -3\frac{1}{24}$ $\therefore b = -1\frac{25}{48}$

5. a) $v = u + at$

$v - u = at$

$\therefore a = \frac{v-u}{t}$

b) $s = \frac{1}{2}(u + v)t$

$2s = (u + v)t$

$\frac{2s}{t} = u + v$

$\frac{2s}{t} - v = u$

c) $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$d^2 - (y_1 - y_2)^2 = (x_1 - x_2)^2$

$\sqrt{d^2 - (y_1 - y_2)^2} = x_1 - x_2$

$x_1 = x_2 + \sqrt{d^2 - (y_1 - y_2)^2}$

d) $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\frac{2S_n}{n} = 2a + (n - 1)d$

$\frac{2S_n}{n} - (n - 1)d = 2a$

$\frac{S_n}{n} - \frac{1}{2}(n - 1)d = a$

e) $(x - a)^2 + (y - b)^2 = r^2$

$(y - b)^2 = r^2 - (x - a)^2$

$y - b = \sqrt{r^2 - (x - a)^2}$

$-b = \sqrt{r^2 - (x - a)^2} - y$

$b = y - \sqrt{r^2 - (x - a)^2}$

6. a) $x + 3 = 2^3$

$x + 3 = 8$

$x = 5$

b) $3^x + 6 = 87$

$3^x = 81$

$3^x = 3^4$

$x = 4$

c) $m^3 - 2 = 341$

$m^3 = 343$

$m = \sqrt[3]{343}$

$m = 7$

d) $8^n + 2 = 6$

$2^{3n} = 4$

$2^{3n} = 2^2$

$\therefore 3n = 2$

$n = \frac{2}{3}$

e) $p^3 - 7 = 118$

$p^3 = 125$

$p = \sqrt[3]{125}$

$p = 5$

f) $\frac{q^4}{3} = 27$

$q^4 = 81$

$q = \sqrt[4]{81}$

$q = 3$

g) $7^r = 1$

$7^r = 7^0$

$r = 0$

h) $6^t + 2 = 38$

$6^t = 36$

$6^t = 6^2$

$t = 2$

i) $4^v = \frac{1}{2}$

$2^{2v} = 2^{-1}$

$2v = -1$

$v = -\frac{1}{2}$

j) $w^{-3} = \frac{1}{125}$

$w^{-3} = 5^{-3}$

$w = 5$

7. a) Train 1: $d = 100t + 68$ (d = distance; t = time in hours)

Train 2: $d = 120t + 30$

$\therefore 100t + 68 = 120t + 30$

$\therefore 38 = 20t$

$\therefore t = 1.9 \; or \; 1 \; hour \; and \; 54 \; minutes$

Therefore the trains will pass each other after 1 hour and 54 minutes.

b) $2^x = 16$

$2^x = 2^4$

$\therefore x = 4$

Therefore Amy's money will grow to 16 times the amount it is now in 4 x 3 = 12 years.

c) $h = d^2 - 15d + 54$ where $h = 0$

$0 = d^2 - 15d + 54$

$0 = (d - 9)(d - 6)$

$d = 9 \quad or \quad d = 6$

∴ The ball is thrown 6m from the tree and lands 9m from the tree.

d) $Cost = R50 + R30 \times number\ of\ days$

$R230 = R50 + R30 \times n$

$R180 = R30 \times n$

$6 = n$

∴ Sipho rented the tuxedo for 6 days.

e) $Cost = R5 \times number\ of\ keyrings + R2000$

$Profit = R14 \times number\ of\ keyrings.$

To start making a profit, your profit must be more than or equal to your cost:

$\therefore 5n + 2000 = 14n$

$\therefore 2000 = 9n$

$\therefore n = 222\frac{2}{9}$

∴ To make a profit the company needs to sell 223 or more keyrings.

f) Total cost = R400 x 24 = R9600

Total paid = Original Cost (1 + interest x number of years)

$R9\,600 = R5999\big(1 + interest \times (24 \div 12)\big)$

$1.600 = 1 + interest \times 2$

$0.6 = interest \times 2$

$0.3 = interest$

∴ The company is charging Vusi 30% interest

g) Length and breadth of the box is x

$\therefore (x + 8)(x + 7) = 210$

$x^2 + 8x + 7x + 56 - 210 = 0$

$x^2 + 15x - 154 = 0$

$(x + 22)(x - 7) = 0$

$x = -22 \quad or \quad x = 7$

$x \neq -22$ as it is impossible to have negative length.

∴ The length of the box is 7mm + 8mm = 15mm and the width is 7mm + 7mm = 14mm.

h) $3 \times burgers + 2 \times cans = R138$ $5 \times burgers + 6 \times cans = R254$ 2

$2c = 138 - 3b$

$c = 69 - \frac{3}{2}b$ 1

Subs 1 into 2

$\therefore 5b + 6\left(69 - \frac{3}{2}b\right) = 254$

$5b + 414 - 9b = 254$

$-4b = -160$

$\therefore b = 40$

Subs back into 1

$c = 69 - \frac{3}{2}(40)$

$c = 69 - 60$

$c = 9$

∴ A burger costs R40 and a can costs R9 and together they cost R49.