Worksheet 11 Memorandum: Geometry of 2D Shapes

Grade 9

1. a) right-angled triangle - *A triangle that has one angle equal to 90°*

   b) isosceles triangle – *A triangle that has two sides equal, and two angles corresponding to those sides also equal.*

   c) equilateral triangle – *A triangle where all three sides are equal and all three angles are equal.*

   d) scalene triangle – *A triangle where no sides are equal and no angles are equal. All three sides and angles are different.*

   e) rectangle – *A quadrilateral with 4 sides, opposite pairs of sides are equal and parallel, and all four angles are equal to 90°*

   f) square – *A quadrilateral where all four sides are equal, opposite sides are parallel, and all four angles are equal to 90°*

   g) rhombus – *A quadrilateral where all four sides are equal, opposite sides are parallel, and opposite angles are equal.*

   h) trapezium – *A quadrilateral where one pair of sides is parallel.*

   i) parallelogram – *A quadrilateral where opposite sides are parallel and equal. Opposite angles are equal.*

   j) kite – *A quadrilateral where two pairs of adjacent sides are equal, and the diagonals intersect at 90°*

   k) similar - *Two shapes are similar when either all of their corresponding angles are equal, or their sides are in proportion.*

   l) congruent – *Two shapes are congruent when they are identical. To prove congruency, we need to either prove:*

      i) SSS – all 3 sides are equal

      ii) SAS – two sides and an included angle are equal

      iii) AAS – two angles and a corresponding side are equal

      iv) RHS – Right angle, hypotenuse and side are equal.

2. a) Rhombus b) Equilateral triangle

   c) Rectangle d) Parallelogram

   e) Kite f) Isosceles triangle.

   g) Scalene triangle h) Trapezium

3. Look at each of the shapes below and name them based on their properties

   a) Right-angled triangle b) Square

   c) Trapezium d) Rhombus

4. What is the sum of interior angles for a

   a) 180° b) 360°

5. a) $A\hat{B}C = B\hat{E}C + E\hat{C}B$ Exterior angle = sum of opp. Int angles.

   $A\hat{B}C = 90° + 34°$

   $\therefore A\hat{B}C = 124°$

   b) $x = A\hat{B}C = 124°$ opp. Angles of parallelogram are equal

   c) In ΔABC and ΔADC.

   1. $A\hat{B}C = A\hat{D}C$ proven above
   2. $B\hat{C}A = C\hat{A}D$ BC || AD
   3. $B\hat{A}C = A\hat{C}D$ AB || DC

   ∴ΔBCA |||ΔDAC

   d) In ΔABC and ΔADC

   1. AB = DC opp. Sides of parallelogram are equal
   2. BC = AD opp. Sides of parallelogram are equal
   3. AC is common

   ∴ ΔABC ≡ ΔDAC.

e) Trapezium, opposite sides parallel, no sides are equal.

f) No, because AD is parallel to BC and BC and EC meet at C. Therefore, EC cannot be parallel to AD.

g) $D\hat{A}B + A\hat{D}C = 180°$ Co-int angles AB ∥ DC

$\therefore D\hat{A}B = 180° - 124°$ $A\hat{D}C = 124°$ proven above

$\therefore D\hat{A}B = 56°$

6. a) 6 squares

b) ΔBAD, ΔADC, ΔDCB, ΔCBA, ΔAED, ΔDEC, ΔCEB, ΔBEA (any 4)

c) ΔDEA, ΔAEB, ΔBEC, ΔCED, ΔDCB, ΔCBA, ΔBAD, ΔADC, ΔHMI, ΔGLF, ΔDEJ, ΔDEN, HCI, ΔBIF, ΔGAF, ΔDHG (any 4)

d) BIDF

e) In ΔDGF and ΔDHI

1. DG = DH midpoints of a square
2. DF = DI midpoints of a square from same point.
3. HI = GF midpoint to midpoint on square equal.

∴ΔDFG ≡ ΔDIH

f) LMNJ, JNIF, GLJK, LMIF, LHIF (any 2)

7. a) $A\hat{G}F + F\hat{G}D = 180°$ Angles on a straight line

$\therefore A\hat{G}F = 180° - 120°$ $F\hat{G}D = 120°$ given

$\therefore A\hat{G}F = 60°$

b) In ΔAGF, $A\hat{G}F = 60°$, CF = AD (regular hexagon), so that means FG = AG (because G is the midpoint of CF and AD. Thus ΔAGF is an isosceles triangle and $A\hat{F}G = G\hat{A}F$.

But, $A\hat{F}G + G\hat{A}F + A\hat{G}F = 180°$ Angles in $\Delta = 180°$

So, $2A\hat{F}G = 180° - 60°$ $A\hat{F}G = G\hat{A}F$.

$\therefore 2A\hat{F}G = 120°$

$And\ \therefore A\hat{F}G = 60°$

That means that $A\hat{F}G = G\hat{A}F = 60°$

And this proves that ΔAGF is an equilateral triangle.

c) In FEDG

We know that FE = ED — Regular hexagon

And that FG = GD — Proved above

A hexagon has a sum of interior angles equal to (6 – 2)x 180° = 720°

That means that each angle in the hexagon is 720° ÷ 6 = 120°

Then $F\hat{E}D = F\hat{G}D = 120°$

Now, FG = AF (proved above)

And AF = FE — Regular hexagon

So that means FE = ED = DG = GF

So FEDG is a rhombus because all 4 sides are equal and opposite angles are equal.

d) ED || FC — EDGF is a rhombus (proved above)

∴ EDCF is a trapezium as it has one set of side parallel.

e) In AFDC

AF = DC — Regular hexagon, sides are equal

$G\hat{F}A = 60°$ — Proven above

And ΔGFD is an isosceles with GF = GD (proved above)

So $G\hat{F}D = F\hat{D}G$

And $G\hat{F}D + F\hat{D}G + 120° = 180°$

So, $2G\hat{F}D = 180° - 120°$

$\therefore G\hat{F}D = 30°$

Now 30° + 60° = 90° $G\hat{F}D + A\hat{F}G = A\hat{F}D = 90°$

And similarly, for $F\hat{A}C$, $A\hat{C}D$ and $C\hat{D}F$

∴ AFDC is a rectangle; opposite sides are equal and all four angles are 90°

f) In AFHI

Because FG = FA, it cannot be equal to FH, or $A\hat{F}G$ would be 90° (we proved that it is actually 60°), this means that AFHI is a rectangle.

8. There are 11 squares