Investigation: Exponents

Grade 9 Maths

Important Calculator Skills for this exercise:

Prime factors are the prime numbers that divide perfectly into another number. To find a prime factor using your Sharp calculator type in the number, e.g., 72, and press \( \boxed{=} \). Now press \( \boxed{2ndF} \) \( \boxed{\text{Exp}} \). You should see this on your screen:

\[
\begin{array}{c}
\boxed{72}= \\
\boxed{2^3 \times 3^2}
\end{array}
\]

To type in the exponents on the calculator you can use the \( \boxed{\text{yx}} \) button.

E.g., Type in \( 3^3 \times 3^4 \) by pressing \( \boxed{3} \) \( \boxed{\text{yx}} \) \( \boxed{3} \) \( \boxed{\times} \) \( \boxed{3} \) \( \boxed{\text{yx}} \) \( \boxed{4} \) \( \boxed{=} \)

\[
\begin{array}{c}
\boxed{3^3 \times 3^4} = \\
\boxed{2187}
\end{array}
\]

Law 1

1. Complete the following table using your calculator:
   The first line has been done as an example:

   \[
   \begin{array}{|c|c|c|c|c|}
   \hline
   a^m & a^n & a^m \times a^n & \text{Prime factors} & a^{m+n} \\
   \hline
   2^2 & 2^3 & 32 & 32 = 2^5 & 2^{2+3} = 32 \\
   3^3 & 3^3 & & & \\
   2^4 & 2^5 & & & \\
   3^4 & 3^5 & & & \\
   5^3 & 5^7 & & & \\
   a^2 & a^4 & & & \\
   2^x & 2^y & & & \\
   \hline
   \end{array}
   \]
2. Can you give a general rule for what is happening in the table above?
   a) State this rule in words.
   b) Complete the rule in symbols:
      \[ a^m \times a^n = a^{\ldots + \ldots} \]

3. Can you think of any examples that disprove this rule?

Law 2

1. Complete the following table using your calculator:

The first line has been done as an example:

<table>
<thead>
<tr>
<th>( a^m )</th>
<th>( a^n )</th>
<th>( a^m \div a^n )</th>
<th>Prime factors</th>
<th>( a^{m-n} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2^5</td>
<td>2^3</td>
<td>4</td>
<td>4 = 2^2</td>
<td>2^{5-3} = 4</td>
</tr>
<tr>
<td>3^3</td>
<td>3^3</td>
<td>\text{---}</td>
<td>\text{---}</td>
<td>\text{---}</td>
</tr>
<tr>
<td>2^6</td>
<td>2^5</td>
<td>\text{---}</td>
<td>\text{---}</td>
<td>\text{---}</td>
</tr>
<tr>
<td>3^{14}</td>
<td>3^5</td>
<td>\text{---}</td>
<td>\text{---}</td>
<td>\text{---}</td>
</tr>
<tr>
<td>5^7</td>
<td>5^3</td>
<td>\text{---}</td>
<td>\text{---}</td>
<td>\text{---}</td>
</tr>
<tr>
<td>a^6</td>
<td>a^4</td>
<td>\text{---}</td>
<td>\text{---}</td>
<td>\text{---}</td>
</tr>
<tr>
<td>2^x</td>
<td>2^y</td>
<td>\text{---}</td>
<td>\text{---}</td>
<td>\text{---}</td>
</tr>
</tbody>
</table>

2. Can you give a general rule for what is happening in the table above?
   a) State this rule in words.
   b) Complete the rule:
      \[ a^m \div a^n = a^{\ldots - \ldots} \]

3. Can you think of any examples that disprove this rule?
For this section you need to know how to type this \((3^3)^2\) into the calculator.

Press \(3\) \(\times y^x\) \(3\) \(\Rightarrow\)

Then \(\frac{1}{y^x} 2\)

(the Sharp calculator will automatically add these brackets for you).

And \(\Rightarrow\)

**Law 3**

1. Complete the following table using your calculator:
   The first line has been done as an example:

<table>
<thead>
<tr>
<th>(a^m)</th>
<th>n</th>
<th>((a^m)^n)</th>
<th>Prime factors</th>
<th>(a^{mxn})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2^2)</td>
<td>3</td>
<td>((2^2)^3 = 64)</td>
<td>(64 = 2^6)</td>
<td>(2^{2\times3} = 64)</td>
</tr>
<tr>
<td>(3^3)</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2^4)</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3^4)</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5^3)</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a^2)</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2^x)</td>
<td>y</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Can you give a general rule for what is happening in the table above?
   a) State this rule in words.
   b) Complete the rule:

   \[(a^m)^n = a^{mxn}\]
3. Can you think of any examples that disprove this rule?

4. Does this rule apply to every number in the brackets?

5. Is \((2 \times 3^2)^2\) the same as \((2)^2 \times (3^2)^2\)?

6. What can we say about the above? Can you think of any examples that disprove this rule?

Law 4

On your calculator go to the table mode.

On the Sharp EL-W535SA press \(\text{MODE} \rightarrow 2\).

Now type in an X (press \(\text{RCL}\) twice).

Then press the exponent button and put in a zero.

Press equals until you reach the table.

1. What do you notice about the answer column?

2. Is there any place in the answer column where this value changes?
3. Complete this table:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^0$</td>
<td></td>
</tr>
<tr>
<td>$4^0$</td>
<td></td>
</tr>
<tr>
<td>$5^0$</td>
<td></td>
</tr>
<tr>
<td>$2^0$</td>
<td></td>
</tr>
<tr>
<td>$x^0$</td>
<td></td>
</tr>
</tbody>
</table>

4. What is the general rule?

5. Complete the rule:

$$a^0 = ....$$

**Why does this happen?**

Well, let’s go back to Law 2: $a^m ÷ a^n = a^{m-n}$

If we have that $a^m ÷ a^m$ it gives us $a^{m-m} = a^0$

Just like: $3^3 ÷ 3^3 = 3^{3-3} = 3^0$

But we know that anything divided by itself is 1.

On your calculator press [OFF] twice.

Type in [RCL] [RCL] [a/b] [RCL] [RCL]. Your screen should look like this:

Press equals until you reach your table.

6. What do you see?

7. Do you think this proves the rule we discussed above?

8. Can you think of any examples that disprove this rule?
Law 5

On your calculator press \( \boxed{\text{ON/C}} \) twice (to go back to the first screen of your table mode).

Now type in \( \boxed{\text{RCL} \ \text{RCL} \ \boxed{\text{X}^{-1}} \ \boxed{1}} \), so that you have:

\[
\begin{array}{c}
X \\
-1
\end{array}
\]

Then press \( \boxed{\text{=}} \) four times.

Use your arrow keys to scroll through the table.

1. Complete the table below. The first row has been done as an example for you. (Hint: You will need to fill in the answer column first, then go back to normal mode (press \( \boxed{\text{HOME}} \)) and find the equivalent fraction of the decimal.)

<table>
<thead>
<tr>
<th>( X )</th>
<th>( \text{ANS} )</th>
<th>( \text{Fraction} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.5</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What do you notice about the \( X \) and the fraction?

3. Complete the rule:

\( x^{-1} = \) _____

4. Can you think of examples that disprove this rule?
Now, press \( \text{ON/C} \) twice again. This time, type in:

\[
\frac{1}{\text{X} - 1}
\]

Then press \( \text{=} \) four times.

5. Complete the table below. The first row has been done as an example for you.

<table>
<thead>
<tr>
<th>X</th>
<th>( \frac{1}{X^{-1}} )</th>
<th>ANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( \frac{1}{2^{-1}} )</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. What do you notice about \( \frac{1}{X^{-1}} \) and ANS?

7. Complete the rule:

\[
\frac{1}{a^{-1}} =
\]

8. Can you think of any examples that disprove this rule?